Probabilistic Model Checking of Randomised Distributed Protocols using PRISM

Dave Parker

University of Birmingham

VPSM PhD School, Copenhagen, October 2006

Part II

Tool Support: PRISM

Overview

- Tool support for probabilistic model checking
- The PRISM tool
  - functionality, features, resources
  - modelling language
  - property specification
  - tool demo
  - efficient symbolic implementations
- Related work/research topics

Motivation

- Complexity of PCTL model checking
  - generally polynomial in model size (number of states)
- State space explosion problem
  - models for realistic case studies are typically huge
- Clearly tool support is required
- Benefits:
  - fully automated process
  - high-level languages/formalisms for building models
  - visualisation of quantitative results

Probabilistic model checkers

- PRISM (this talk) – DTMCs, MDPs, CTMCs + rewards
- ETMCC/MRMC – DTMCs, CTMCs + reward extensions
- LiQuor – LTL verification for MDPs (Probmela language)
- RAPTURE - prototype for abstraction/refinement of MDPs
- Simulation-based probabilistic model checking:
  - APMC, Ymer (both based on PRISM language)
- CSL model checking for CTMCs: APNN-Toolbox, SMART
- Multiple formalism/tool solutions: CADP, Möbius

The PRISM tool

- PRISM: Probabilistic symbolic model checker
  - developed at the University of Birmingham, since approx. 1999
  - free, open source
  - versions for Linux, Unix, Mac OS X, Windows

- Construction of models:
  - DTMCs, MDPs , CTMCs + costs/rewards
- Verification of:
  - PCTL, CSL + extensions + costs/rewards

- www.cs.bham.ac.uk/~dxp/prism

PRISM - Functionality

- Constructs three types of probabilistic models:
  - DTMCs, MDPs, CTMCs
  - also: PTAs with digital clocks by manual translation
  - augmented with costs/rewards
- The PRISM language – high-level model description language
- PRISM simulator - generate model traces for debugging, etc.
- Variety of import/export functionality:
  - model output: text files, Dot graphs, Matlab, ETMCC/MRMC
  - model import: text files
  - other input formalisms via language translation: PEPA, CSP
  - direct connections to other tools: APMC, ProVer/Ymer

PRISM - Functionality

- Supports verification of:
  - PCTL (for DTMCs, MDPs), CSL (for CTMCs)
  - plus “quantitative” extensions
  - cost/reward-based properties

- Powerful, flexible implementation
  - efficient symbolic (BDD-based) implementations
  - multiple computation engines
  - wide range of model analysis methods
  - sampling-based computation (discrete-event simulation)

PRISM - Functionality

- Graphical user interface
  - model/property editor
  - easy automation of verification experiments
  - graphical visualisation of results
  - debugging tool: simulation engine

- Command-line version
  - same underlying verification engines
  - useful for scripting, batch jobs

Getting PRISM + Other Resources

- PRISM website: www.cs.bham.ac.uk/~dxp/prism
  - tool download: binaries, source code
  - online example repository (40+ case studies)
  - online documentation
  - support: help forum, bug tracking, feature requests
    - hosted on Sourceforge
  - related publications, links

PRISM modelling language

- Simple, state-based language for DTMCs/MDPs/CTMCs
  - based on Reactive Modules [Alur/Henzinger]
- Modules (system components, composed in parallel)
- Variables (finite-valued, local or global)
- Guarded commands (labelled with probabilities/rates)
- Synchronisation (CSP-style) + process-algebraic operators (parallel composition, action hiding/renaming)

[send] (s=2) -> p_{loss} : (s'=3)&(lost'=lost+1) + (1-p_{loss}) : (s'=4);

action guard probability update probability update

PRISM language example

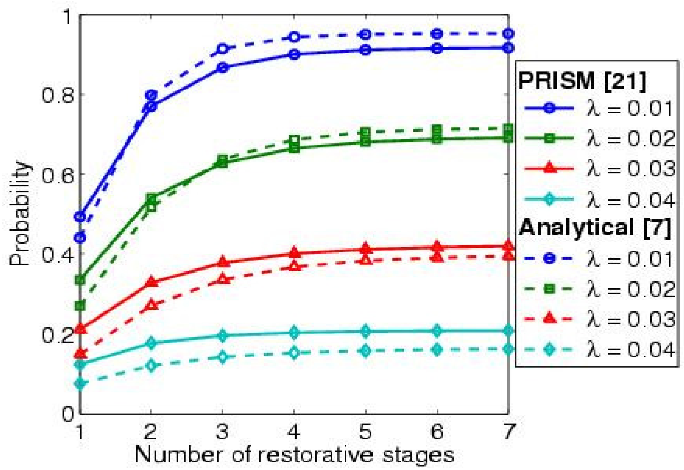
```
// hermans self-stabilisation algorithm [Her90]

dtmc // algorithm is synchronous

module process1 // first of N=5 symmetric processes

    x1 : [0..1];  // one bit per process; xi=x(i-1) means process i has a token

    [step] (x1=x5) -> 0.5 : (x1'=0) + 0.5 : (x1'=1);

    [step] !x1=x5 -> (x1'=x5);

endmodule

// add further processes through renaming
module process2 = process1 [ x1=x2, x5=x1 ] endmodule
module process3 = process1 [ x1=x3, x5=x2 ] endmodule
module process4 = process1 [ x1=x4, x5=x3 ] endmodule
module process5 = process1 [ x1=x5, x5=x4 ] endmodule

// can start in any possible configuration
init true endinit

// cost - 1 in each state (expected number of steps)
rewards true : 1; endrewards
```

PRISM – Property specifications

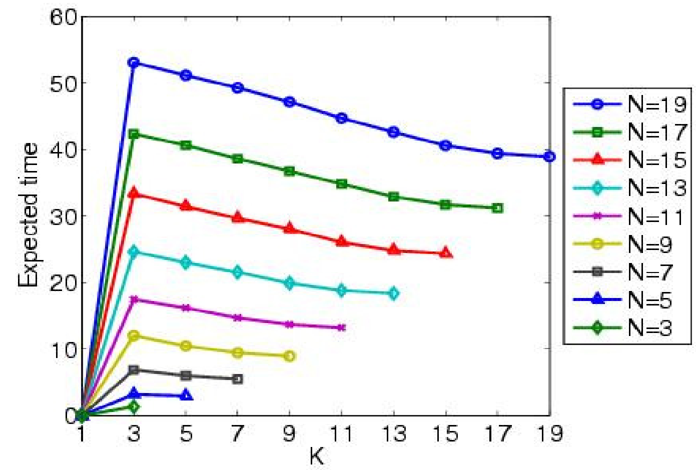
- Based on (probabilistic extensions of) temporal logic
  - incorporates PCTL for DTMCs/MDPs, CSL for CTMCs
- Examples:
  - P<0.001 [F shutdown] - "shutdown eventually occurs with probability at most 0.001"
  - P<0.2 [F[t,t] (deliv_rate < min)] "the probability that the current packet delivery rate has dropped below minimum at time t is less than 0.2"
  - P≥0.95 [!repair U≤200 done] - "with probability 0.95 or greater, the process will successfully complete within 200 hours and without requiring any repairs"
- No counterexamples (error traces) in prob. model checking

PRISM – Property specifications

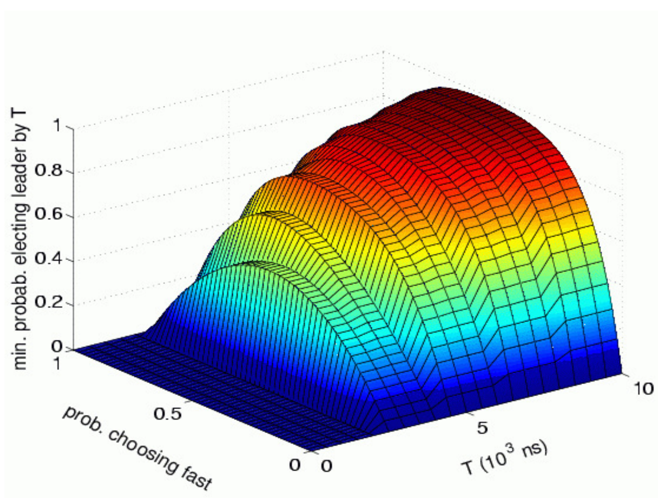
- Focus on quantitative properties, compute actual values
  - P=? [F≤T "shutdown"] - "what is the probability of shutdown occurring within T hours?"
- Best/worst-case scenarios
  - P=? [F "error" {"init"}{max}] - "what is the worst-case error probability over all possible initial configurations?"
  - Pmin=? [!end2 U end1] - "what is the minimum probability of process 1 finishing before process 2, over all possible schedulings of the processes?"
- Experiments – ranges of model/property parameters
  - P=? [F≤T error] for N=1..5, T=1..100
  - identify patterns, trends, anomalies in results



Probability that 10% of gate outputs are erroneous for varying gate failure rates and numbers of stages



Worst-case expected number of steps to stabilise for initial configurations with K tokens amongst N processes



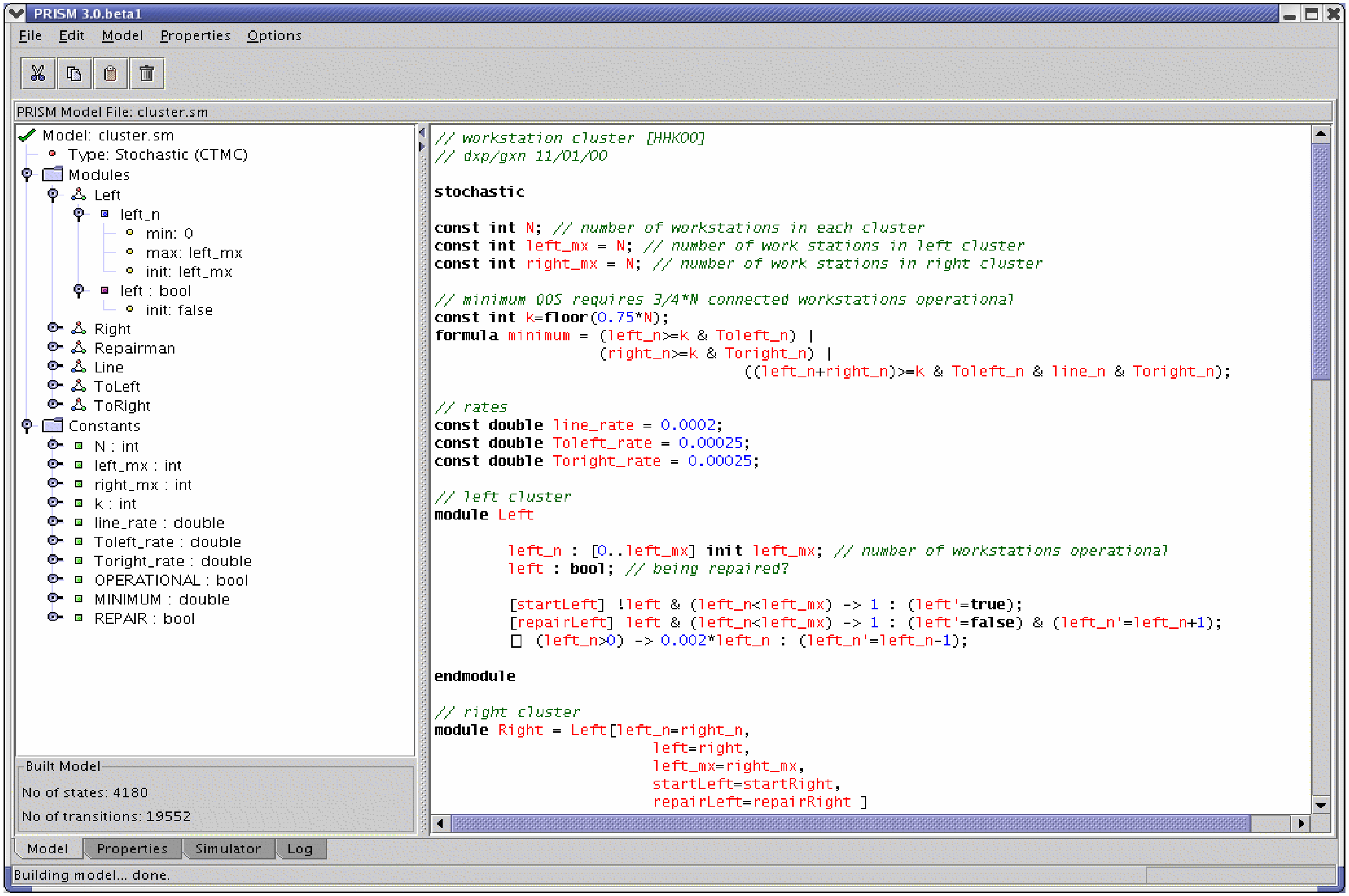
Optimum probability of leader election by time T for various coin biases

Cost- and reward-based properties

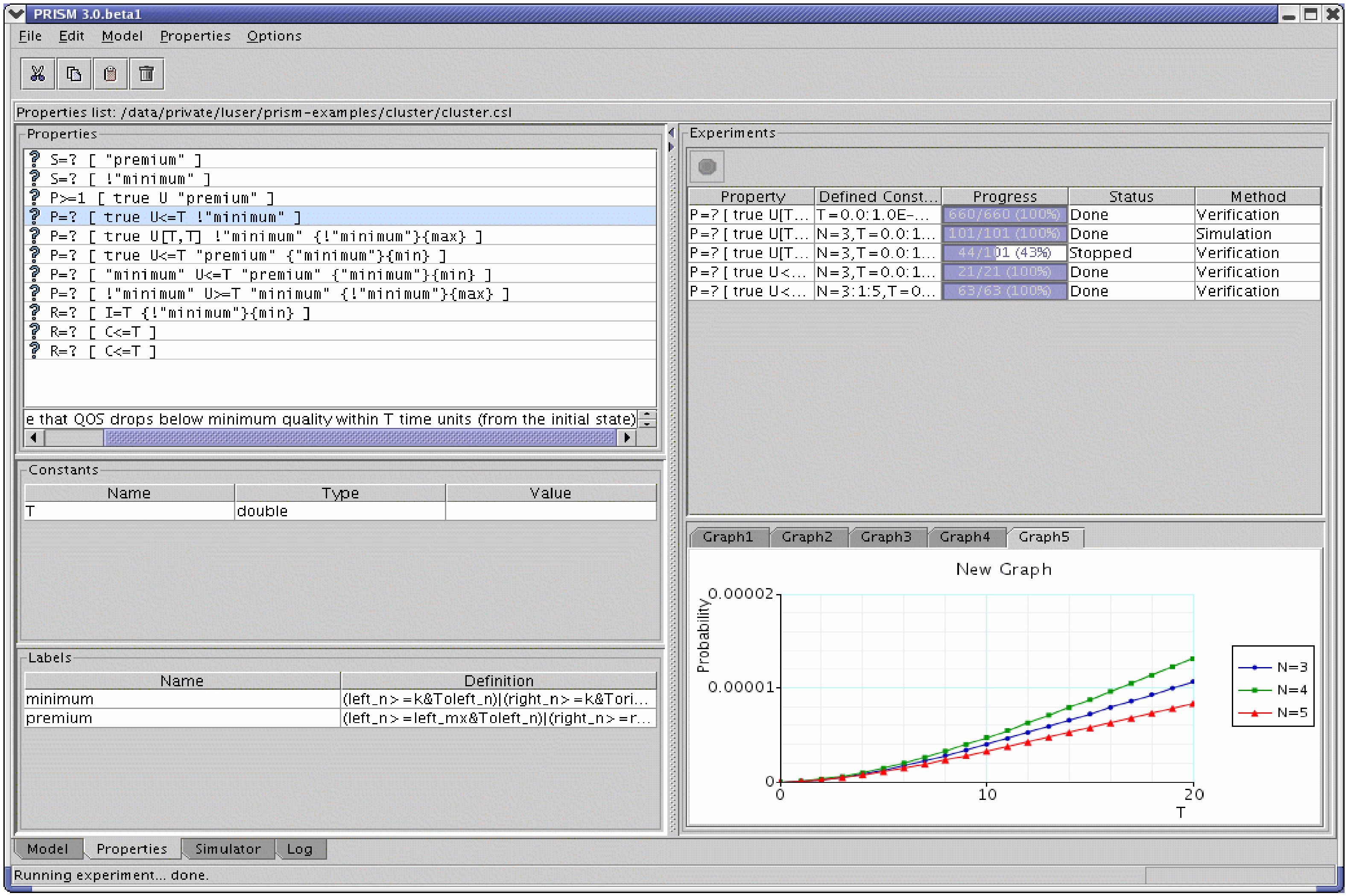
- Costs and rewards
  - real-valued quantities assigned to states/transitions
- Instantaneous – state-based measures
  - current queue size, number of operational channels, ...
  - “what is the expected size of the message queue at time t?”
  - “what is the long-run expected size of the queue?”
- Cumulative – state or transition (impulse) costs/rewards
  - time, power consumption, messages lost, ...
  - “what is the expected power consumption during the first 2 hours of operation?”
  - “what is the worst-case expected time taken for the protocol to terminate?”

PRISM Demo

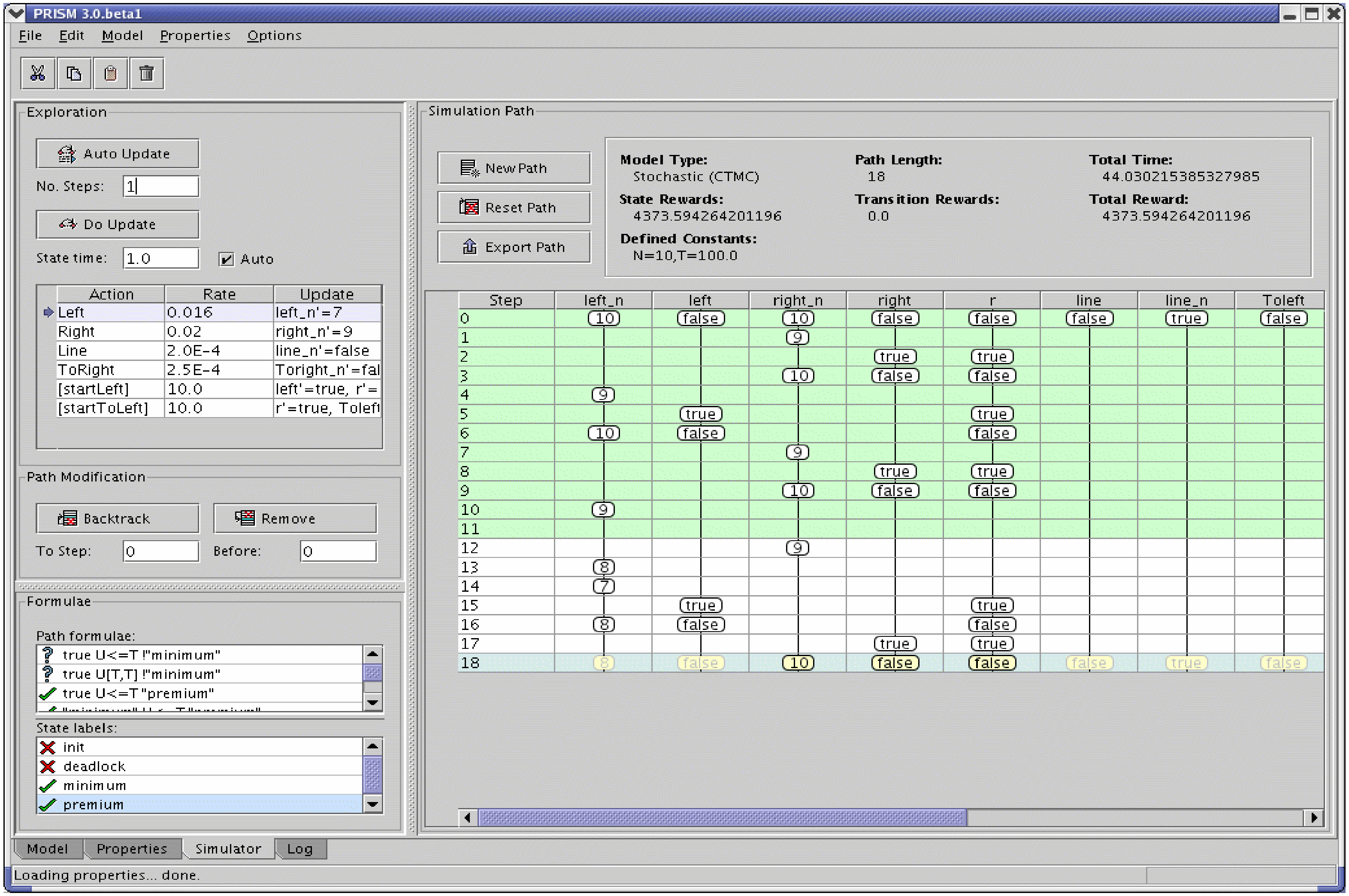
PRISM Screenshots



PRISM Screenshots



PRISM Screenshots



Efficiency - Symbolic techniques

- State space explosion
  - models of real-life systems typically huge
- Symbolic probabilistic model checking
  - data structures based on binary decision diagrams (BDDs)
  - compact storage: exploit model structure and regularity
  - efficient implementation of graph traversal fixed point algorithms
- PRISM: multiple computation engines
  - MTBDDs (BDD extension): storage/analysis of very large models (given structure/regularity), numerical computation can blow up
  - sparse matrices: fastest solution for smaller models ($<10^6$ states), prohibitive memory consumption for larger models
  - hybrid: combine MTBDD storage with explicit storage, ten-fold increase in analysable model size ($\sim 10^7$ states)

Efficiency – Other strategies

- Approximate model checking (see also APMC [LHP06])
  - sampling using Monte Carlo discrete-event simulation
  - performed at modelling language level – better scalability
  - potentially huge number of samples for accurate answers
  - also: statistical hypothesis testing, see e.g. [YS02]
- Parallelisation of model checking
  - distribution of storage/computation across multi-processor machines [KPZM04], networked clusters [ZPK05], grids
  - potentially promising for symbolic approaches – reduced I/O
  - simulation-based computations much easier to distribute

Ongoing research areas

- Abstraction and refinement, see e.g. [DJJL01,KNP06a]
  - construct smaller, abstract model by removing information/variables not relevant to property being checked, iteratively refine abstraction if analysis fails
- Symmetry reduction [DM06, KNP06b]
  - exploit replication of identical components
- Partial order reduction, see e.g. [BGC04], [DN04]
  - exploit commutativity of concurrently executed transitions
- Compositionality, see e.g. [dAHJ01,Che06]
  - analyse full model based on analysis of sub-components

References

- **[BGC04]** C. Baier, M. Grosser, and F. Ciesinski. Partial order reduction for probabilistic systems. In *Proc. QEST'04*, pages 230–239. IEEE Computer Society Press, 2004.
- **[Che06]** L. Cheung. Reconciling Nondeterministic and Probabilistic Choices. Ph.D. thesis, Radboud University of Nijmegen. 2006.
- **[DJJL01]** P. D'Argenio, B. Jeannet, H. Jensen, and K. Larsen. Reachability analysis of probabilistic systems by successive refinements. In *Proc. PAPM/PROBMIV'01*, volume 2165 of *LNCS*, pages 39–56, Springer, 2001.
- **[DN04]** P. D'Argenio and P. Niebert. Partial order reduction on concurrent probabilistic programs. In *Proc. QEST'04*, pages 240–249. IEEE Computer Society Press, 2004.

References

- **[dAHJ01]** L. de Alfaro, T. Henzinger and R. Jhala. Compositional Methods for Probabilistic Systems. In *CONCUR 01: Concurrency Theory, 12th International Conference,* LNCS, Springer-Verlag, 2001.
- **[DM06]** A. Donaldson and A. Miller. Symmetry Reduction for Probabilistic Model Checking using Generic Representatives. In *Proc. 4th International Symposium on Automated Technology for Verification and Analysis (ATVA'06)*, Springer. October 2006.
- **[KNP06a]** M. Kwiatkowska, G. Norman and D. Parker. Game-based Abstraction for Markov Decision Processes. In *Proc. 3rd International Conference on Quantitative Evaluation of Systems (QEST'06)*, pages 157-166, IEEE CS Press. 2006.
- **[KNP06b]** M. Kwiatkowska, G. Norman and D. Parker. Symmetry Reduction for Probabilistic Model Checking. In *Proc. 18th International Conference on Computer Aided Verification (CAV'06)*, volume 4144 of LNCS, pages 234-248, Springer, 2006.

References

- **[KPZM04]** M. Kwiatkowska, D. Parker, Y. Zhang and R. Mehmood. Dual-Processor Parallelisation of Symbolic Probabilistic Model Checking. In Proc. MASCOTS'04, pages 123-130, IEEE CS Press. 2004.
- [**LHP06]** R. Lassaigne, T. Hérault and S. Peyronnet. APMC 3.0: Approximate verification of Discrete and Continuous Time Markov Chains. In *Proc. 3rd International Conference on Quantitative Evaluation of Systems (QEST'06)*, 2006.
- **[YS02]** H. Younes and R. Simmons. Probabilistic verification of discrete event systems using acceptance sampling. In *Proceedings of the 14th International Conference on Computer Aided Verification*, volume 2404 of *LNCS*, pages 223-235, Copenhagen, Denmark, July 2002.
- **[ZPK05]** Y. Zhang, D. Parker and M. Kwiatkowska. A Wavefront Parallelisation of CTMC Solution using MTBDDs. In *Proc. International Conference on Dependable Systems and Networks (DSN'05)*, pages 732-742, IEEE Computer Society Press. 2005.