Parameter synthesis for probabilistic real-time systems

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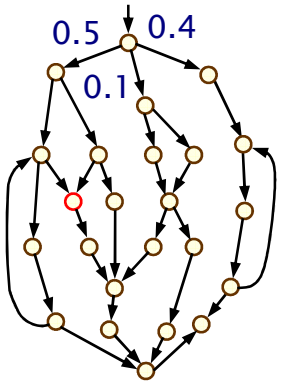
Quantitative (probabilistic) verification

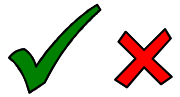
System

Probabilistic model
e.g. Markov chain

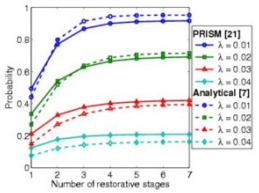


Probabilistic model checker
PRISM

Result



Quantitative results

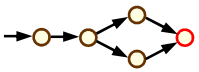


System require-ments

$P_{<0.01}$ [$F^{\leq t}$ fail]

Probabilistic temporal logic specification
e.g. PCTL, CSL, LTL

Counter-example



Historical perspective

- First algorithms proposed in 1980s
  - [Vardi, Courcoubetis, Yannakakis, …]
  - algorithms [Hansson, Jonsson, de Alfaro] & first implementations
- 2000: tools ETMCC (MRMC) & PRISM released
  - PRISM: efficient extensions of symbolic model checking [Kwiatkowska, Norman, Parker, …]
  - ETMCC (now MRMC): model checking for continuous-time Markov chains [Baier, Hermanns, Haverkort, Katoen, …]
- Now mature area, of industrial relevance
  - successfully used by non-experts for many application domains, but full automation and good tool support essential
    - distributed algorithms, communication protocols, security protocols, biological systems, quantum cryptography, planning…
  - genuine flaws found and corrected in real-world systems

Quantitative probabilistic verification

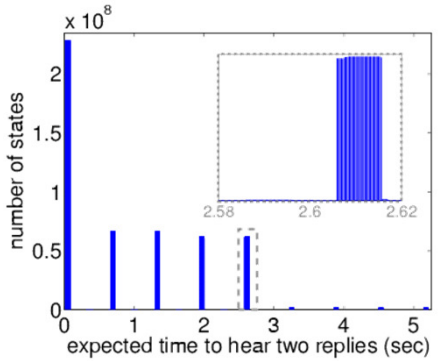
- What's involved
  - specifying, extracting and building of quantitative models
  - graph-based analysis: reachability + qualitative verification
  - numerical solution, e.g. linear equations/linear programming
  - simulation-based statistical model checking
  - typically computationally more expensive than the non-quantitative case
- The state of the art
  - efficient techniques for a range of probabilistic real-time models
  - feasible for models of up to 10^7 states (10^{10} with symbolic)
  - abstraction refinement (CEGAR) methods
  - multi-objective verification
  - assume-guarantee compositional verification
  - tool support exists and is widely used, e.g. PRISM, MRMC

Tool support: PRISM

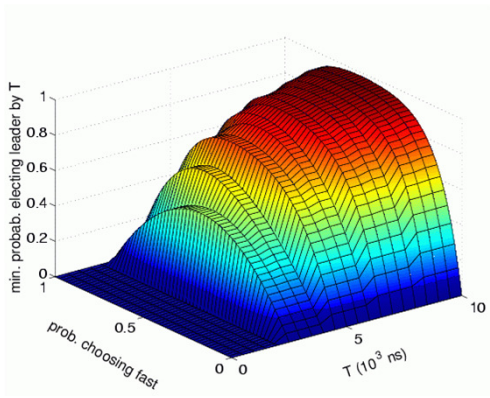
- PRISM: Probabilistic symbolic model checker
  - developed at Birmingham/Oxford University, since 1999
  - free, open source software (GPL), runs on all major OSs
- Support for:
  - models: DTMCs, CTMCs, MDPs, PTAs, SMGs, …
  - properties: PCTL/PCTL*, CSL, LTL, rPATL, costs/rewards, …
- Features:
  - simple but flexible high-level modelling language
  - user interface: editors, simulator, experiments, graph plotting
  - multiple efficient model checking engines (e.g. symbolic)
- Many import/export options, tool connections
  - MRMC, INFAMY, DSD, Petri nets, Matlab, …
- See: http://www.prismmodelchecker.org/

Quantitative verification in action

- Bluetooth device discovery protocol
  - frequency hopping, randomised delays
  - low-level model in PRISM, based on detailed Bluetooth reference documentation
  - numerical solution of 32 Markov chains, each approximately 3 billion states
  - identified worst-case time to hear one message

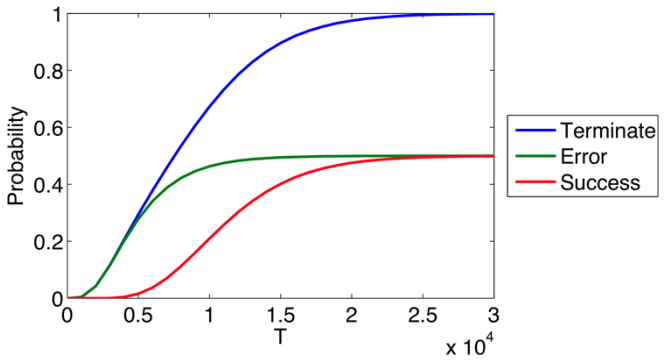


- FireWire root contention
  - wired protocol, uses randomisation
  - model checking using PRISM
  - optimum probability of leader election by time T for various coin biases
  - demonstrated that a biased coin can improve performance

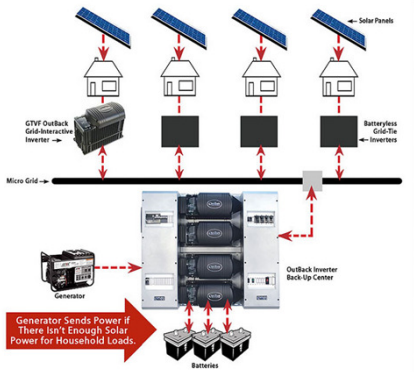


Quantitative verification in action

- DNA transducer gate [Lakin et al, 2012]
  - DNA computing with a restricted class of DNA strand displacement structures
  - transducer design due to Cardelli
  - automatically found and fixed design error, using Microsoft's DSD and PRISM



- Microgrid demand management protocol [TACAS12,FMSD13]
  - designed for households to actively manage demand while accessing a variety of energy sources
  - found and fixed a flaw in the protocol, due to lack of punishment for selfish behaviour
  - implemented in PRISM-games



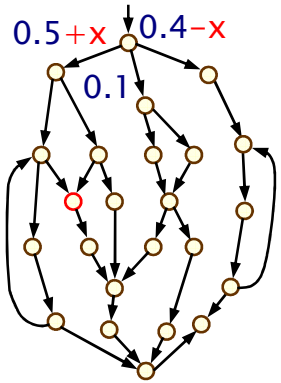
From verification to synthesis…

- Majority of research to date has focused on verification
  - scalability and performance of algorithms
  - extending expressiveness of models and logics
  - real-world case studies
- Automated verification aims to establish if a property holds for a given model
- What to do if quantitative verification fails?
  - counterexamples difficult to represent compactly
- Can we synthesise a model so that a property is satisfied?
  - difficult…
- Simpler variants of synthesis:
  - parameter synthesis
  - controller/strategy synthesis

Quantitative parameter synthesis

System

Parametric model
e.g. Markov chain



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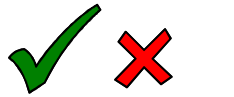
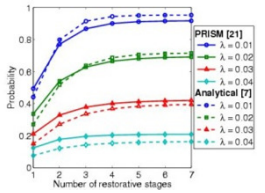
Probabilistic model checker
PRISM PARAM

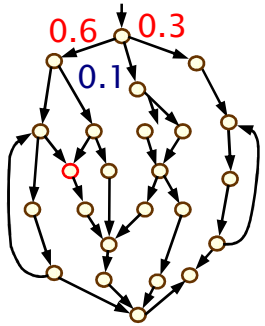
Result



Quantitative results



Concrete model



Parametric model checking in PRISM

- Parametric Markov chain models in PRISM
  - probabilistic parameters expressed as unevaluated constants
  - e.g. const double x;
  - transition probabilities are expressions over parameters, e.g. 0.4 + x
- Properties are given in PCTL, with parameter constants
  - new construct constfilter (min, x1*x2, phi)
  - filters over parameter values, rather than states
- Implemented in 'explicit' engine
  - returns mapping from parameter regions (e.g. [0.2,0.3],[-2,0]) to rational functions over the parameters
  - filter properties used to find parameter values that optimise the function
  - reimplementation of PARAM 2.0 [Hahn et al]

This lecture…

- Parameter synthesis for probabilistic real-time systems
- The parameter synthesis problem we consider
  - given a parametric model and property ϕ
  - find the optimal parameter values, with respect to an objective function O, such that the property ϕ is satisfied, if such values exist
- Parameters: timing delays, rates
- Objectives: optimise probability, reward/volume

Overview

1. Timed automata: find optimal timing delays [EMSOFT2014]
   - solution: constraint solving, discretisation + sampling

2. Probabilistic timed automata: find delays to optimise probability [RP2014]
   - solution: parametric symbolic abstraction-refinement

3. Continuous-time Markov chains: find optimal rates [CMSB2014]
   - solution: constraint solving, uniformisation + sampling

- Focus on practical implementation and real-world applications

1. Optimal timing delays

- Models: networks of timed I/O automata
  - dense real-time
  - extend with parameters on guards
  - synchronise on matching input-output
  - no nondeterminism (add priority and urgency of output)

- Properties: Counting Metric Temporal Logic (CMTL)
  - linear-time, real-valued time bounds
  - event counting in an interval of time, reward weighting

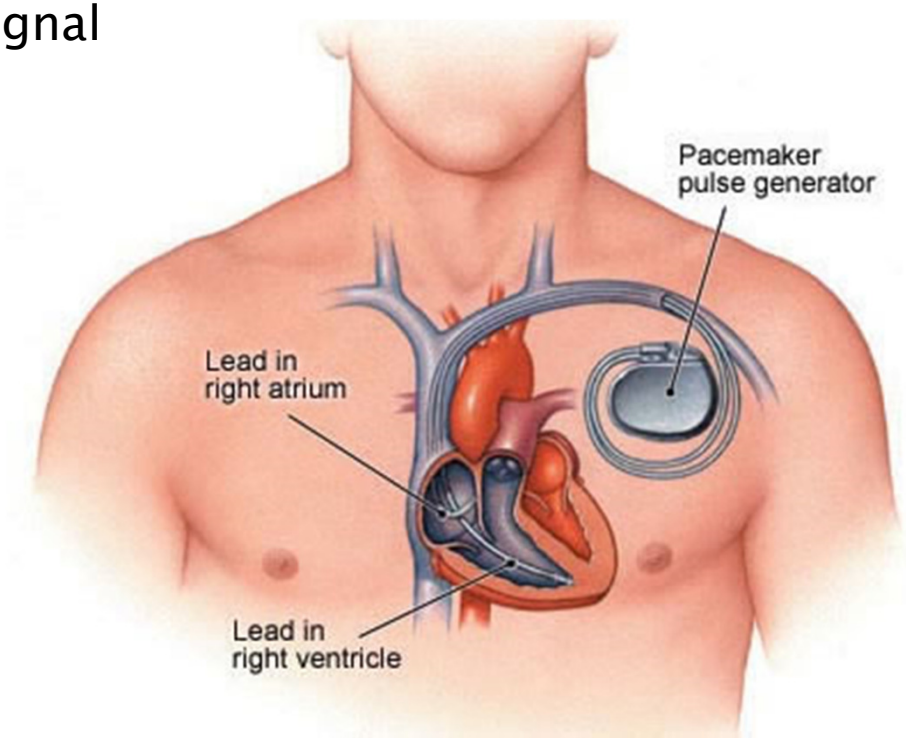
$$\Box^{[0,\tau]}(\#_0^\tau \mathtt{Vget} \geqslant B_1 \wedge \#_0^\tau \mathtt{Vget} \leqslant B_2)$$

$$1 \cdot \#_0^\tau \mathtt{AP} + 2 \cdot \#_0^\tau \mathtt{VP} \leqslant \mathsf{E}$$

Synthesising Optimal Timing Delays for Timed I/O Automata. Diciolla et al. In *14th International Conference on Embedded Software (EMSOFT'14)*, ACM. 2014

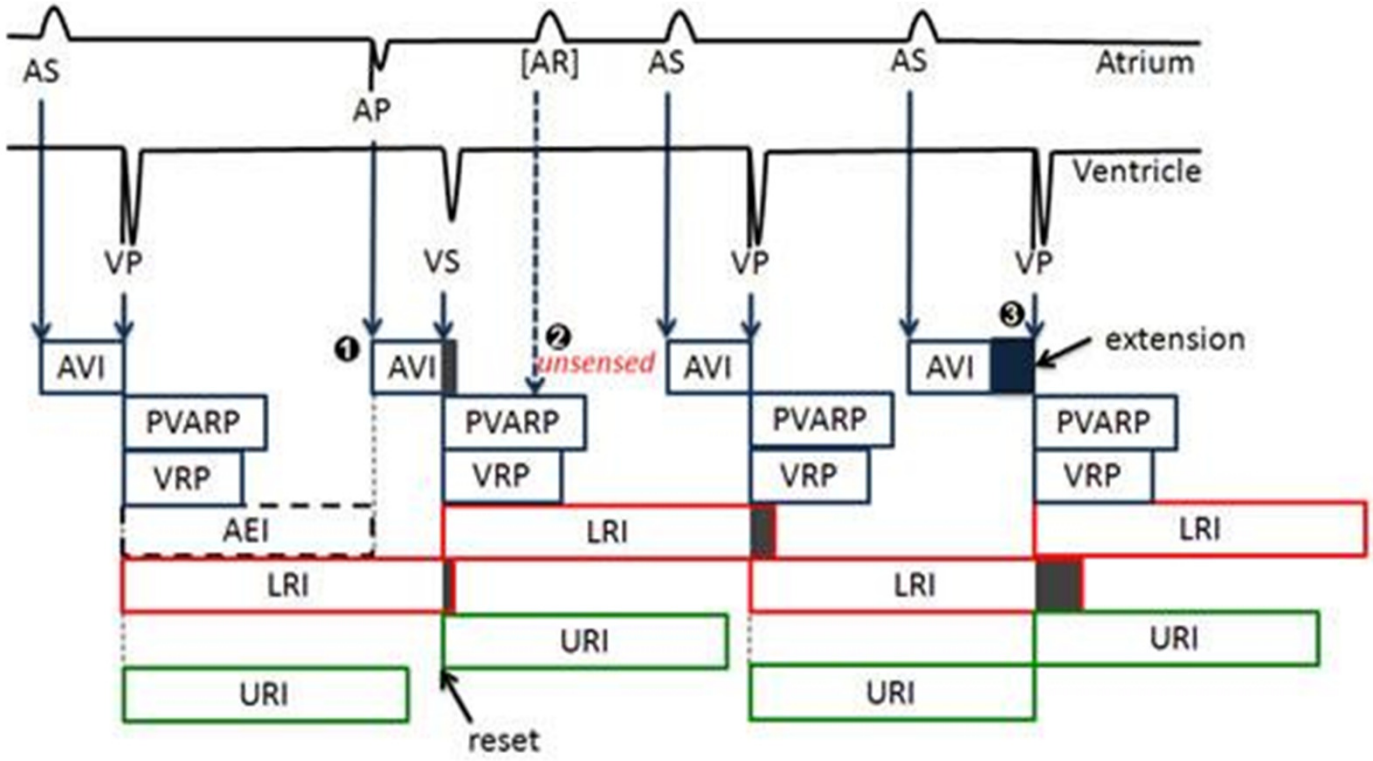
Implantable pacemaker

- How it works
  - reads electrical (action potential) signals through sensors placed in the right atrium and right ventricle
  - monitors the timing of heart beats and local electrical activity
  - generates artificial pacing signal as necessary
- Real-time system!
- Core specification by Boston Scientific
- Basic pacemaker can be modelled as a network of timed automata [Ziang et al]



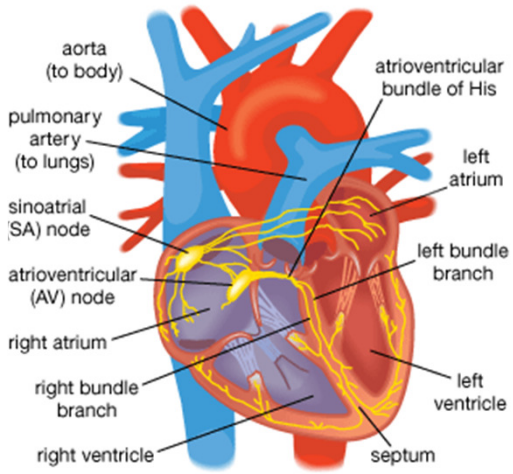
Pacemaker timing cycle

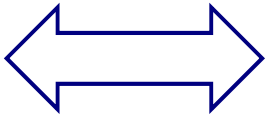
- Atrial and ventricular events



Quantitative verification for pacemakers

- Model the pacemaker and the heart as timed I/O automata
- Compose and verify





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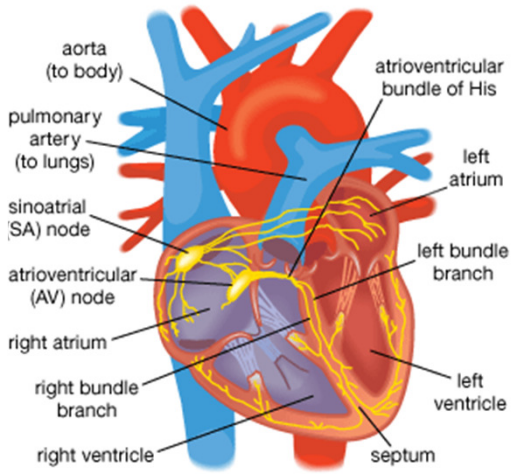
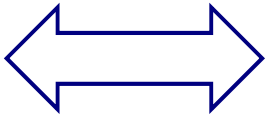
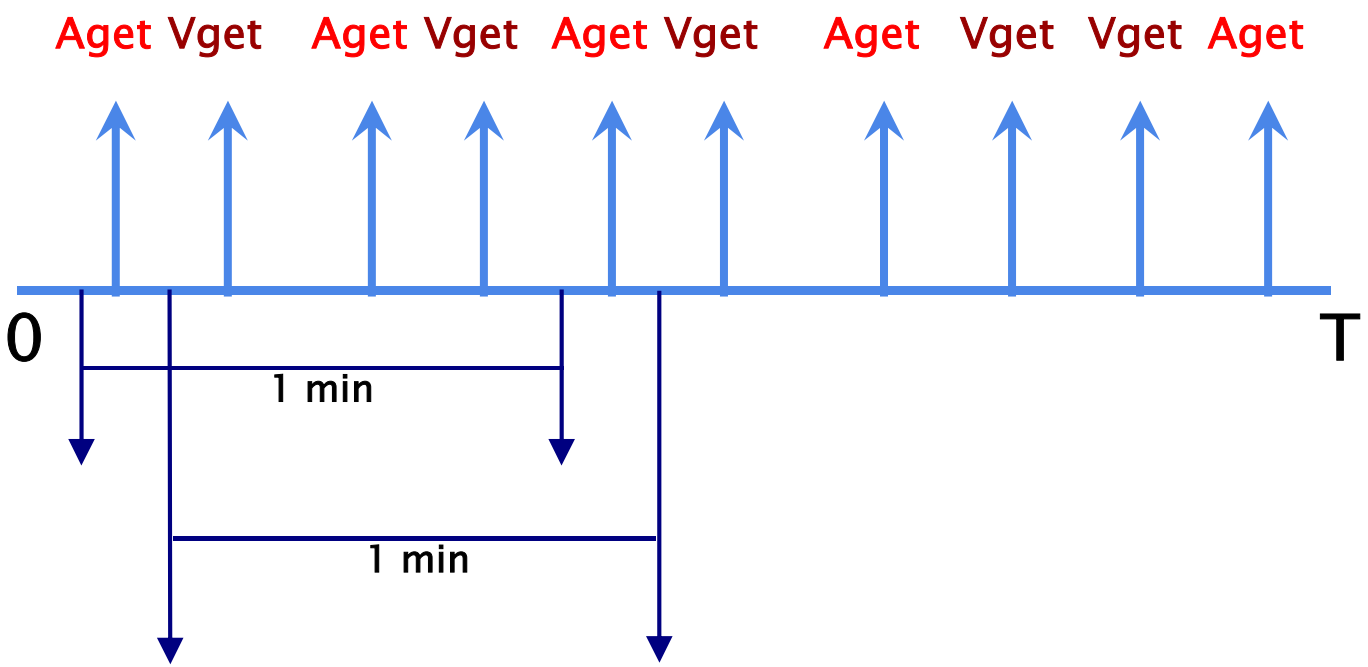




- Can we synthesise (controllable) timing delays to minimise energy, without compromising safety?

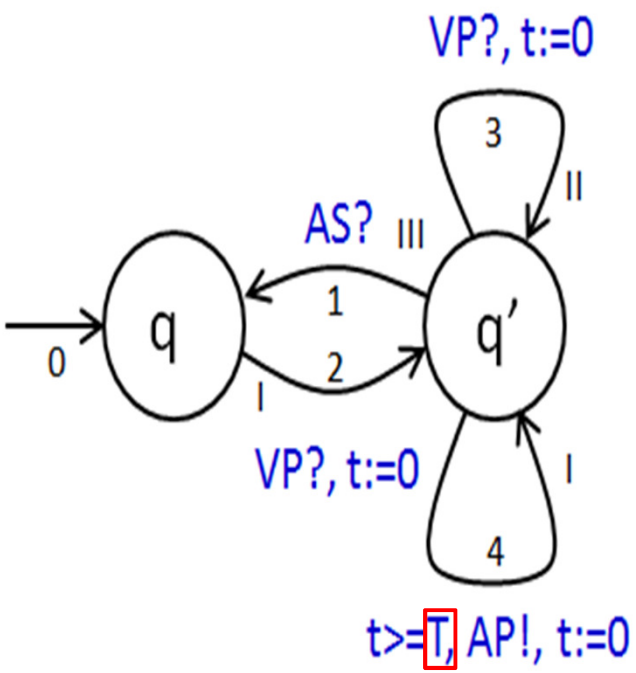
Property patterns: Counting MTL

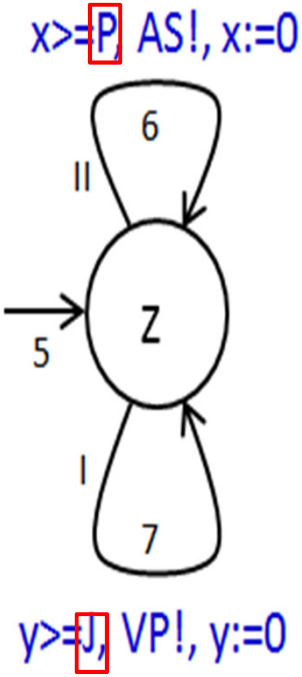


$$\Box^{[0,\tau]}(\#_0^{\tau}\mathtt{Vget} \geqslant B_1 \wedge \#_0^{\tau}\mathtt{Vget} \leqslant B_2)$$

Safety "for any 1 minute window, heart rate is in the interval [60,100]"

Example: timed I/O automata





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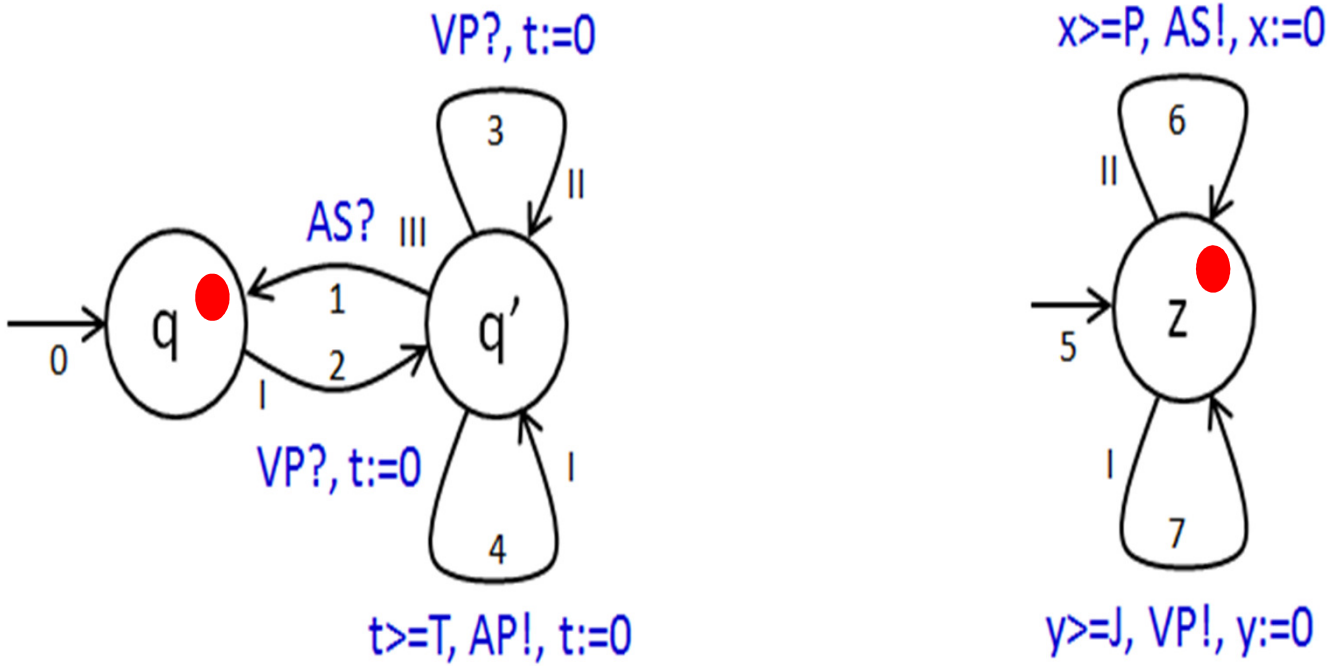
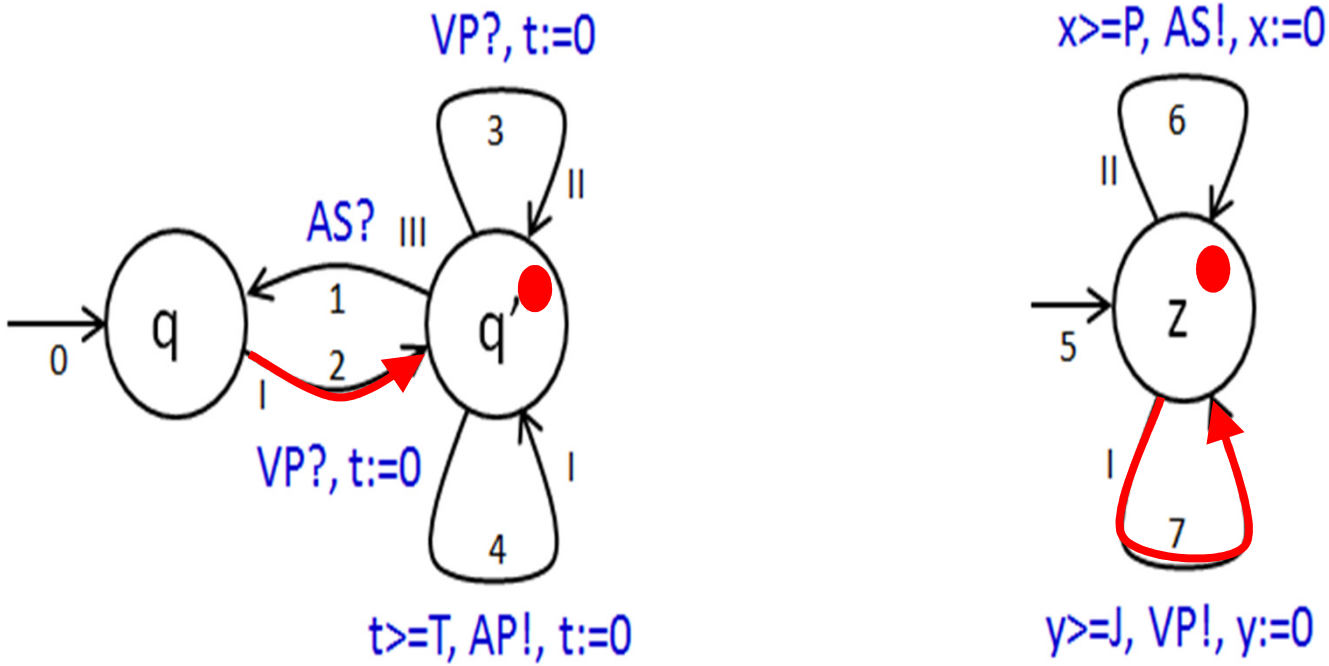


$$\rho = \boxed{(q, z)} \xrightarrow{t_0} (q', z) \xrightarrow{t_1} (q, z)$$

Example: timed I/O automata



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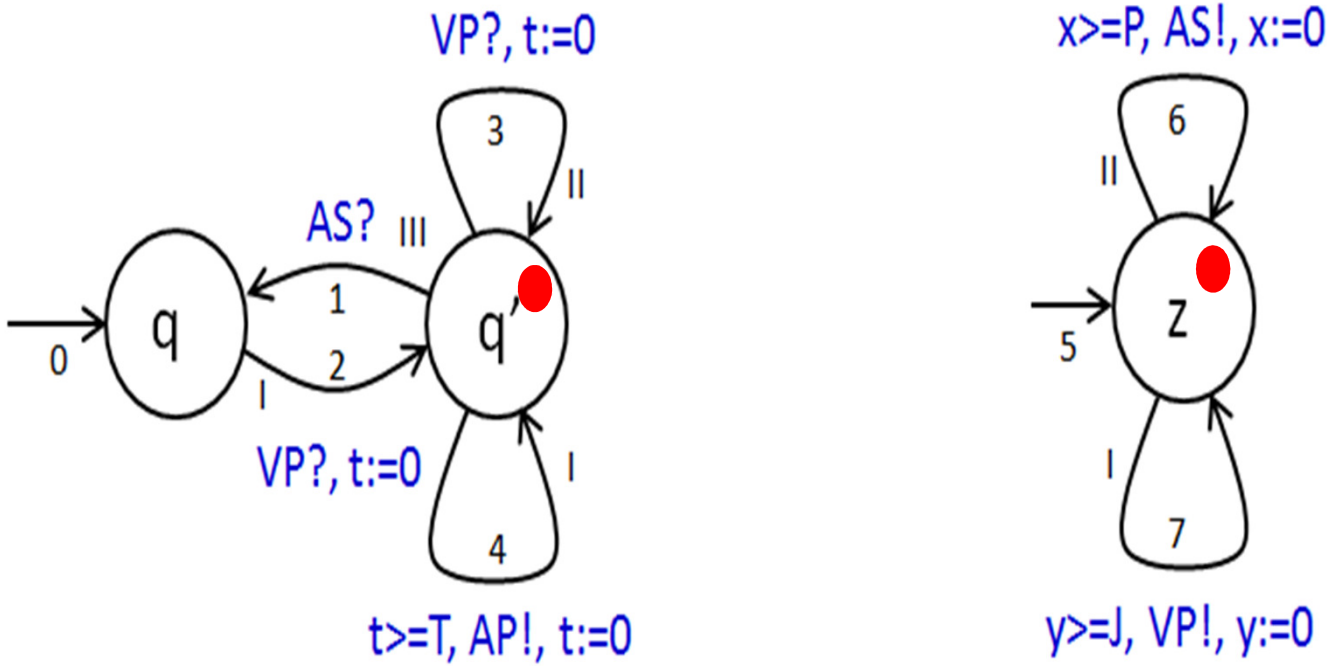

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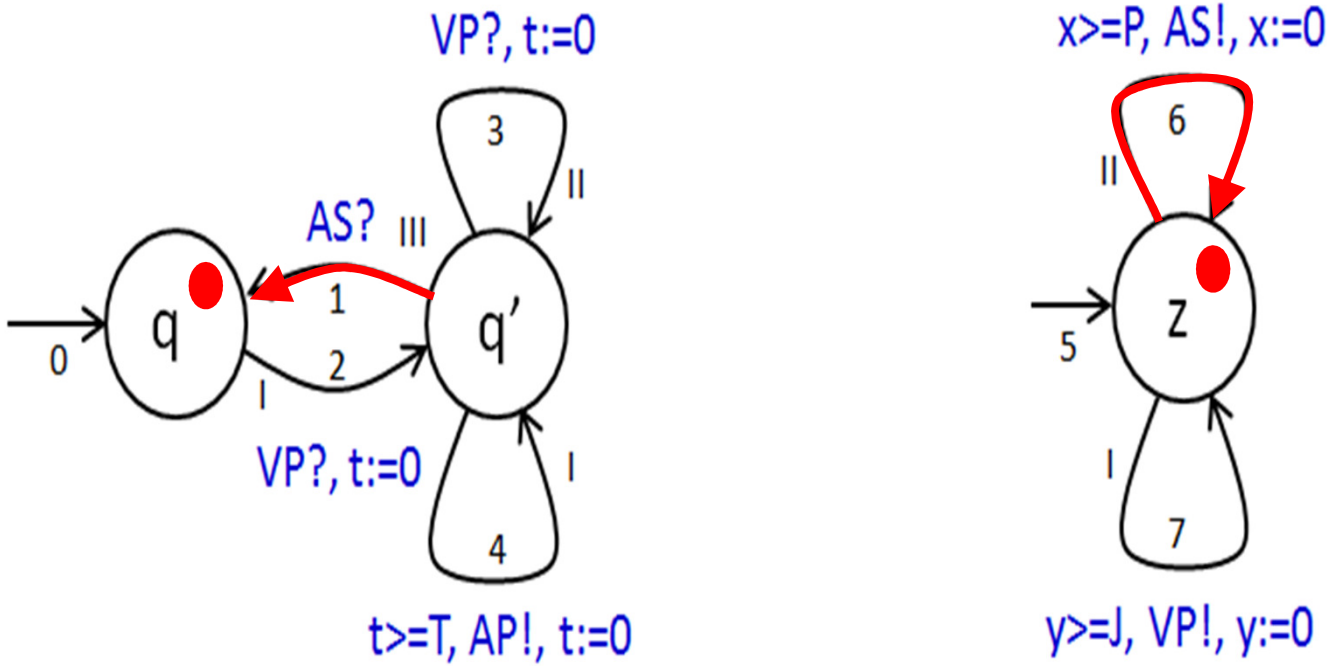
Example: timed I/O automata



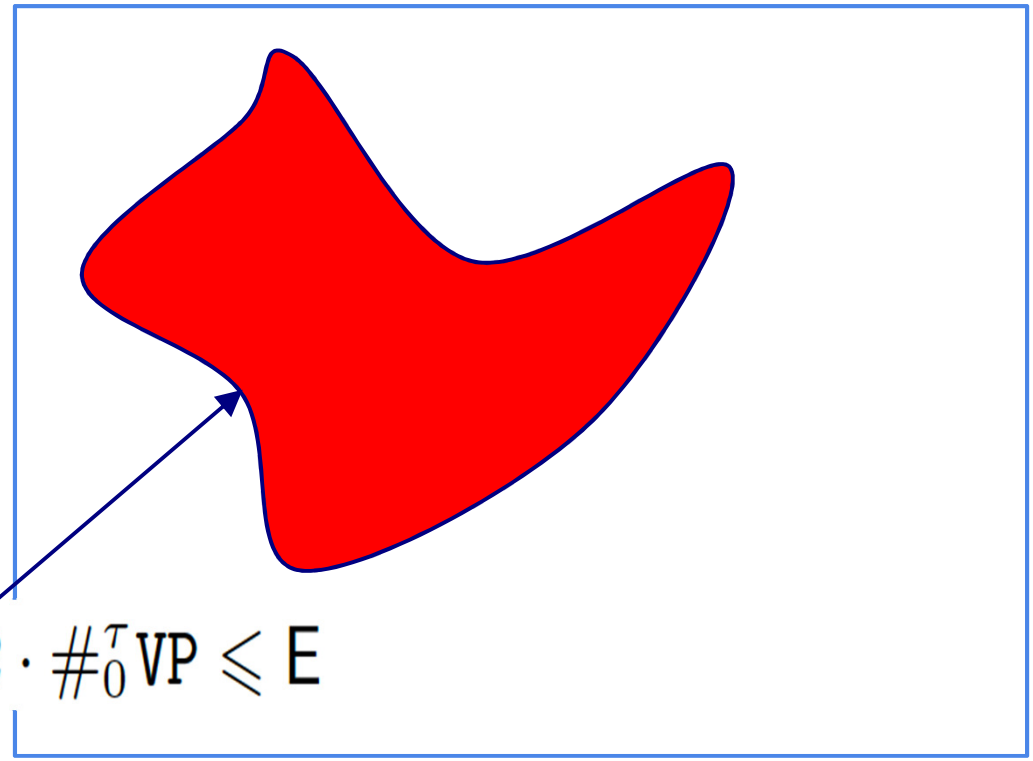
$$\rho = (q, z) \xrightarrow{t_0} (q', z) \xrightarrow{t_1} \boxed{(q, z)}$$

Optimal timing delays problem

- The parameter synthesis problem solved is
  - given a parametric network of timed I/O automata, set of controllable and uncontrollable parameters, CMTL property ϕ and length of path n
  - find the optimal controllable parameter values, for any uncontrollable parameter values, with respect to an objective function O, such that the property ϕ is satisfied on paths of length n, if such values exist
- Consider family of objective functions
  - maximise volume, minimise energy
- Discretise parameters, assume bounded integer parameter space and path length
  - decidable but high complexity (high time constants)

Synthesising Optimal Timing Delays for Timed I/O Automata. Diciolla et al. In *14th International Conference on Embedded Software (EMSOFT'14)*, ACM. 2014

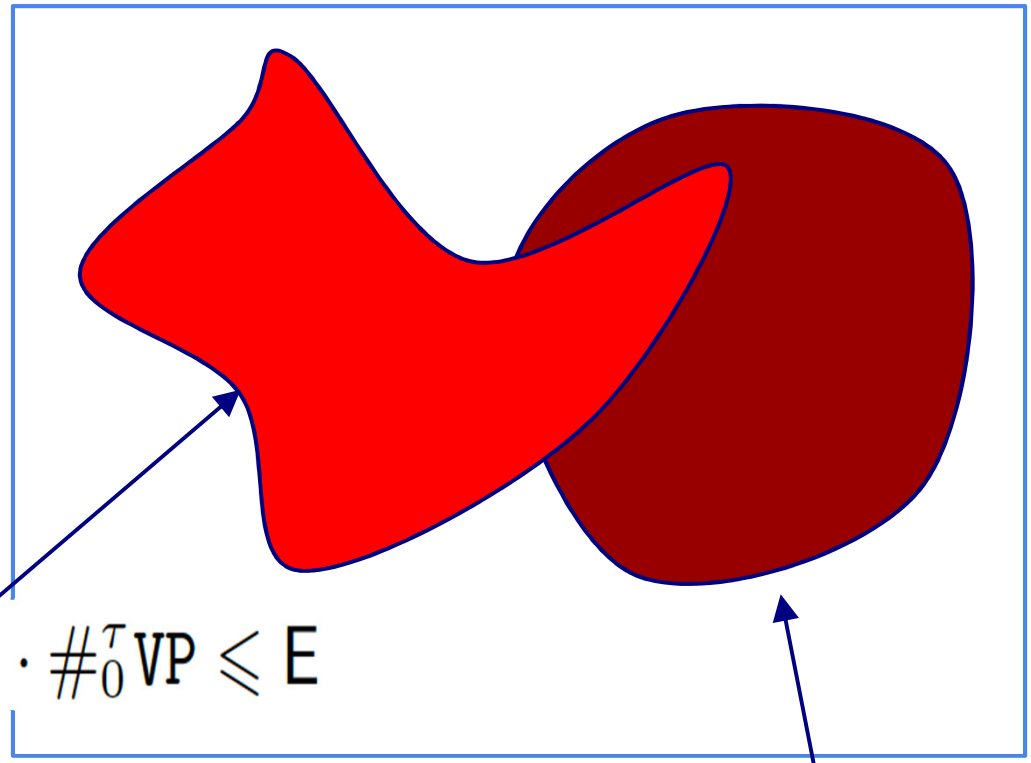
Parameter synthesis



$$1 \cdot \#_0^\tau \mathtt{AP} + 2 \cdot \#_0^\tau \mathtt{VP} \leqslant \mathtt{E}$$

energy

Parameter synthesis



$$1 \cdot \#_0^\tau \mathtt{AP} + 2 \cdot \#_0^\tau \mathtt{VP} \leqslant \mathtt{E}$$

energy

safety

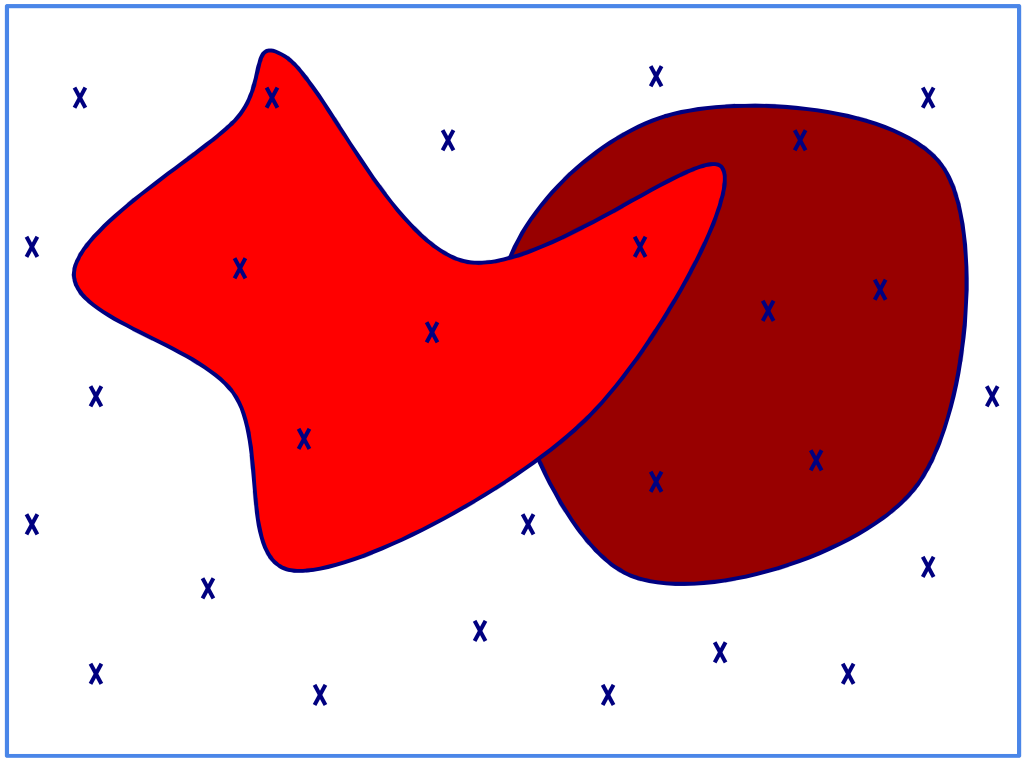
$$\Box^{[0,\tau]} \left(\#_0^\tau \mathtt{Vget} \geqslant B_1 \wedge \#_0^\tau \mathtt{Vget} \leqslant B_2 \right)$$

Our approach

- Constraints generation: all valuations that satisfy property
- Parameter optimisation: select best parameter values

- Sample the domain of the model parameter in order to generate a discrete path

Parameter sampling



Our approach

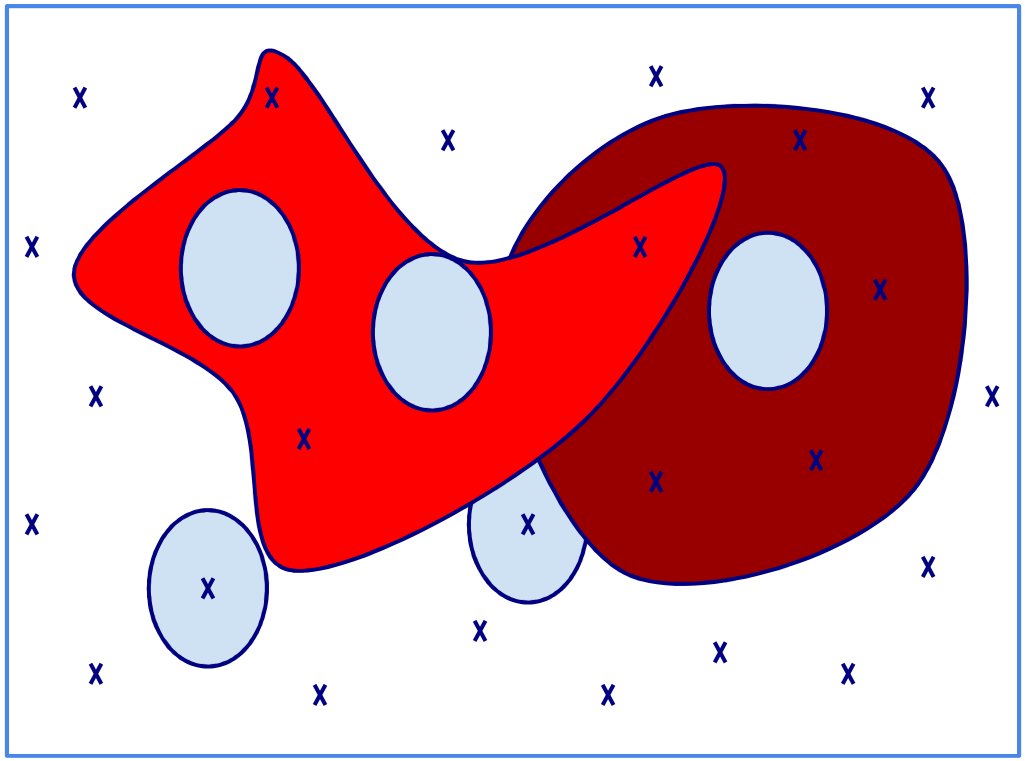
- Constraints generation: all valuations that satisfy property
- Parameter optimisation: select best parameter values

- Sample the domain of the model parameter in order to generate a discrete path

- For each sampled parameter:
  - generate the untimed path
  - generate all inequalities which satisfy the CMTL property

- Advantage: more behaviours can be covered
  - need high coverage, but also need to consider robustness

Constraints generation

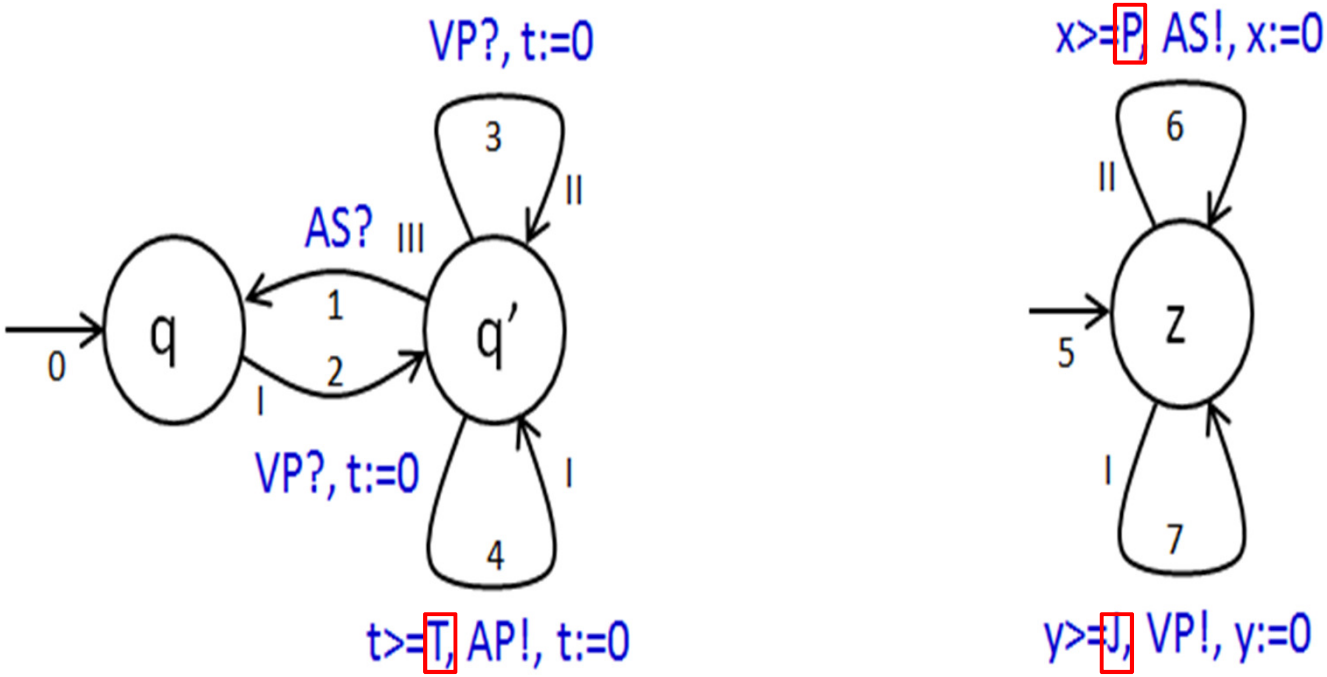


Parameter synthesis algorithm

Require: Network $\mathcal{N}$, formula φ and path length n
Ensure: Formula $\mathcal{S}$

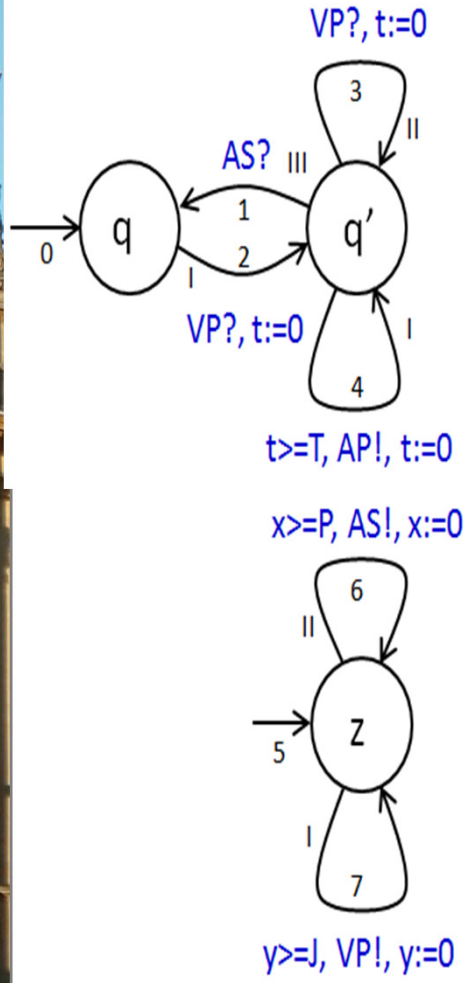
1: **Function** Sat$(\mathcal{N}, \varphi, n)$
2: $\bar{\Gamma} :=$ Sample(Γ)
3: **for** $\vartheta \in \bar{\Gamma}$ **do**
4: **if** $\vartheta \notin \mathcal{S}$ **then**
5: $\rho :=$ Gen_path$(\mathcal{N}, n, \vartheta)$
6: $(\mathcal{S}_\rho, \mathcal{T}) :=$ Path_Constr_Gen$(\mathcal{N}, \rho)$
7: $\mathcal{S}_\varphi :=$ Constr_Gen$(\rho, 0, \varphi, \mathcal{T})$
8: $\mathcal{S} := \mathcal{S} \bigvee (\mathcal{S}_\rho \bigwedge \mathcal{S}_\varphi)$
9: **end if**
10: **end for**
11: **return** $\mathcal{S}$

Back to example…



$$\rho = (q, z)[2, 7](q', z)[4, -](q', z)$$

CMTL property

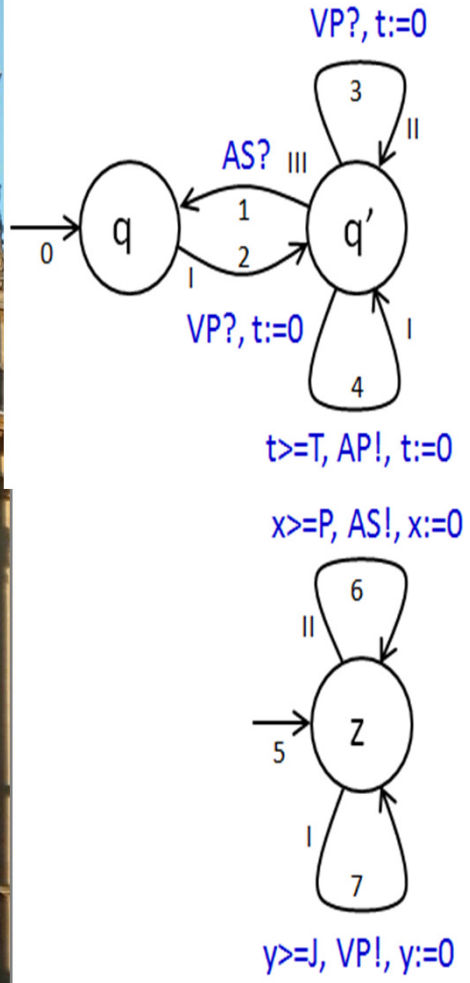


$$\varphi = \#_5^7 \mathsf{VP} \geqslant 1$$

$(t_0 > 5)$	**false**

$$\rho = (q, z)[2, 7](q', z)[4, -](q', z)$$

CMTL property

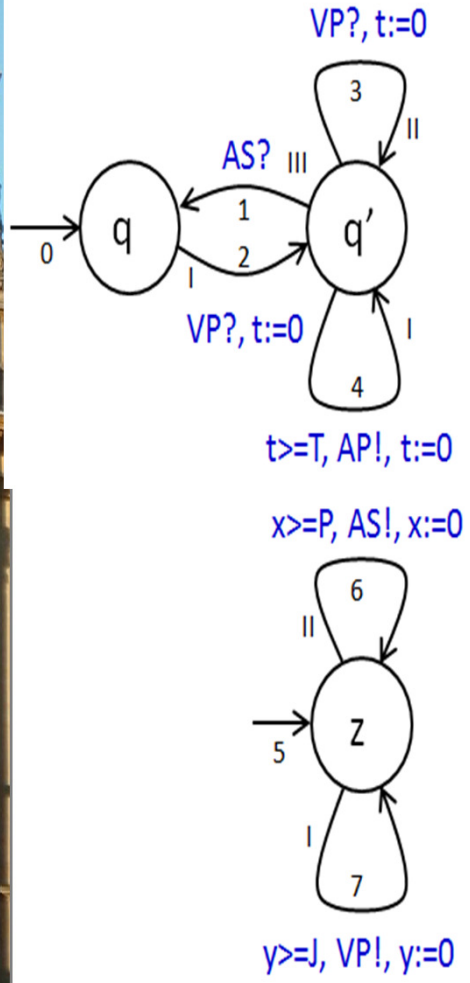


$$\varphi = \#_5^7 \mathsf{VP} \geqslant 1$$

$(t_0 > 5)$	**false**
$(t_0 > 5) \wedge$ $(t_0 + t_1 > 7 \wedge t_0 < 7)$	**true**

$$\rho = (q, z)[2, 7](q', z)[4, -](q', z)$$

CMTL property



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$(t_0 > 5)$	**false**
$(t_0 > 5) \wedge$ $(t_0 + t_1 > 7 \wedge t_0 < 7)$	**true**
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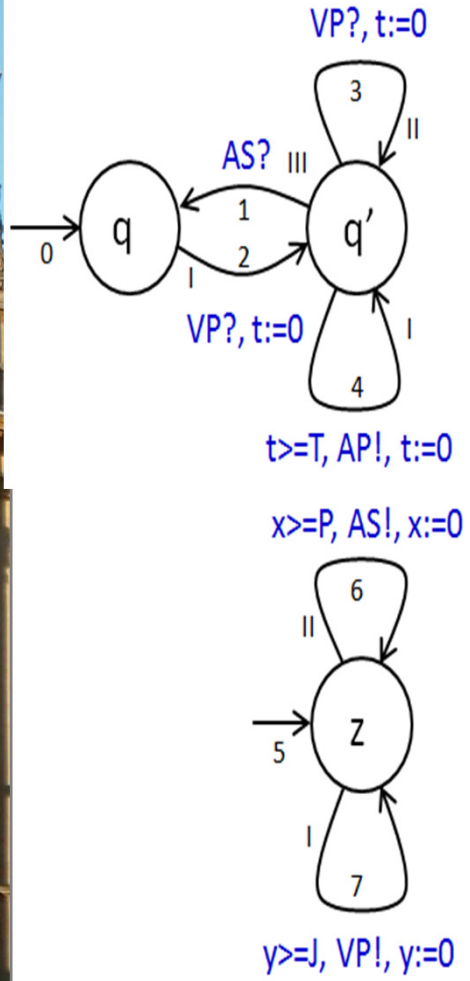
CMTL property



$$\varphi = \#_5^7 \mathsf{VP} \geqslant 1$$

$(t_0 > 5)$	**false**
$(t_0 > 5) \wedge$ $(t_0 + t_1 > 7 \wedge t_0 < 7)$	**true**
$(t_0 + t_1 > 5 \wedge t_0 < 5) \wedge$ $(t_0 + t_1 > 7 \wedge t_0 < 7)$	**false**

$$t_0 = \mathcal{T}[0,7] \text{ and } t_1 = \mathcal{T}[1,4]$$

$$\rho = (q,z)[2,7](q',z)[4,-](q',z)$$

Parameter optimisation

- Have obtained constraints on parameters that satisfy formula
- Maximal

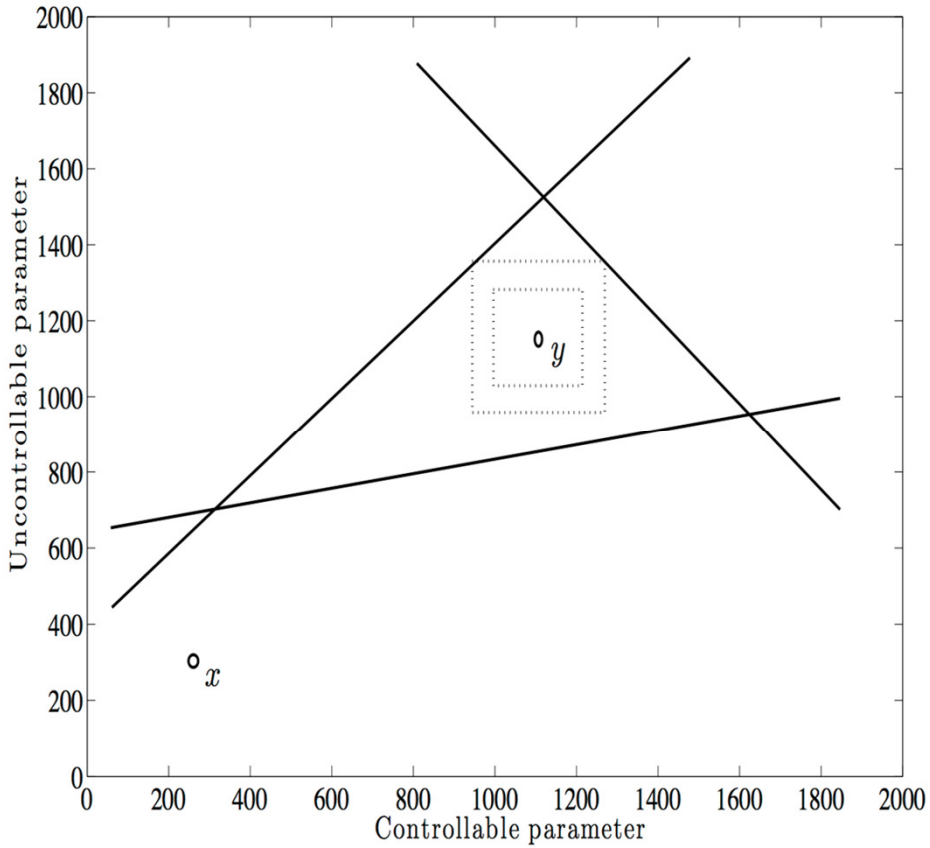
$$\mathrm{opt}_v := \underset{\vartheta_c \in \mathcal{V}(\Gamma_c)}{\mathrm{argsup}} \int_{\vartheta_u \in \mathcal{V}(\Gamma_u), (\vartheta_c, \vartheta_u) \in \mathcal{S}} Distr_{\Gamma_u}(d\vartheta_u)$$

- Robust objective function

$$B_\epsilon(\vartheta) = \{\vartheta' \in \mathcal{V}(\Gamma) \mid ||\vartheta' - \vartheta||_\infty \leqslant \epsilon\},$$

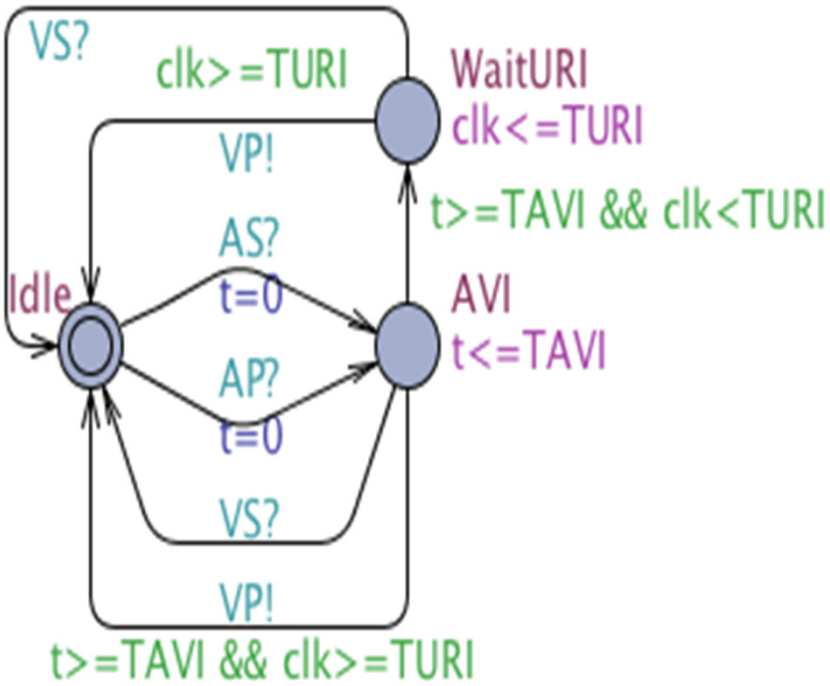
$$\mathrm{opt}_r := \underset{\vartheta_c \in \mathcal{V}(\Gamma_c)}{\mathrm{argsup}} \{\sup_\epsilon \{\epsilon \mid \vartheta_u \in \mathcal{V}(\Gamma_u), B_\epsilon((\vartheta_c, \vartheta_u)) \subseteq \mathcal{S}\}\}$$

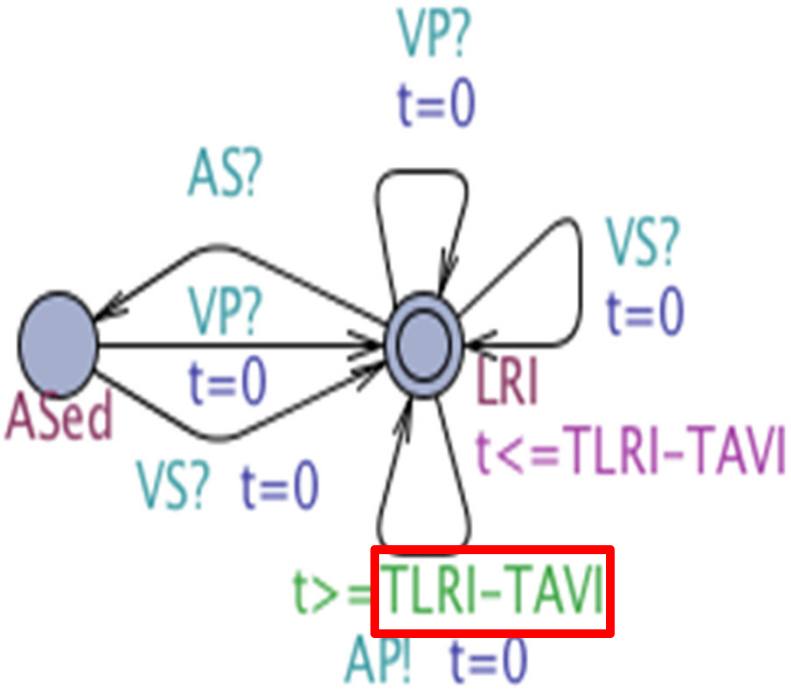
Robust objective function



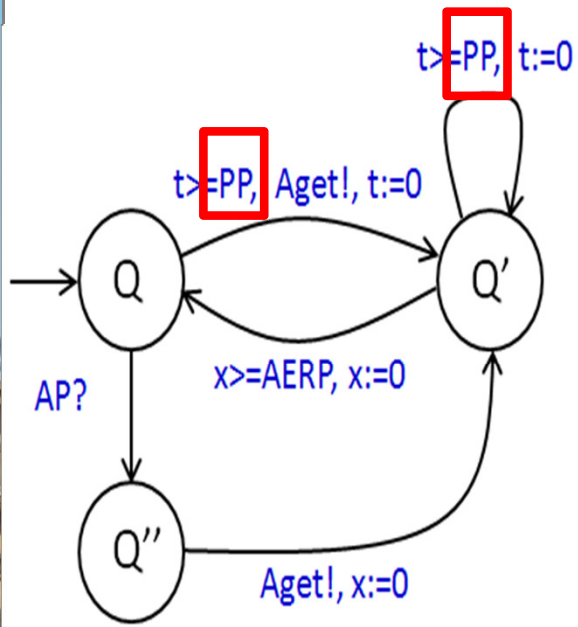
- For each sample point (controllable and uncontrollable)
  - generate path, safety and energy constraints
  - take disjunction, conjuncted with parameter bounds

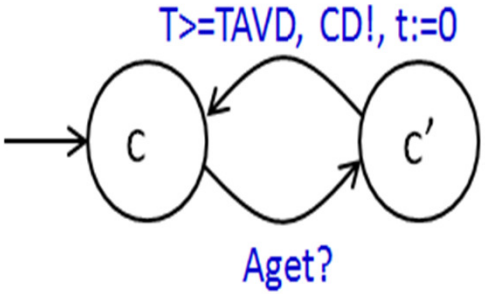
Pacemaker timed I/O automata model

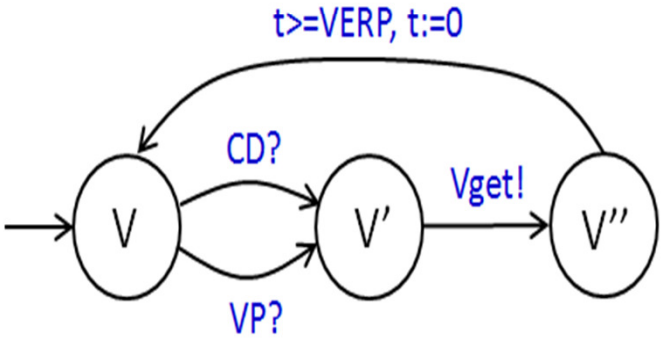




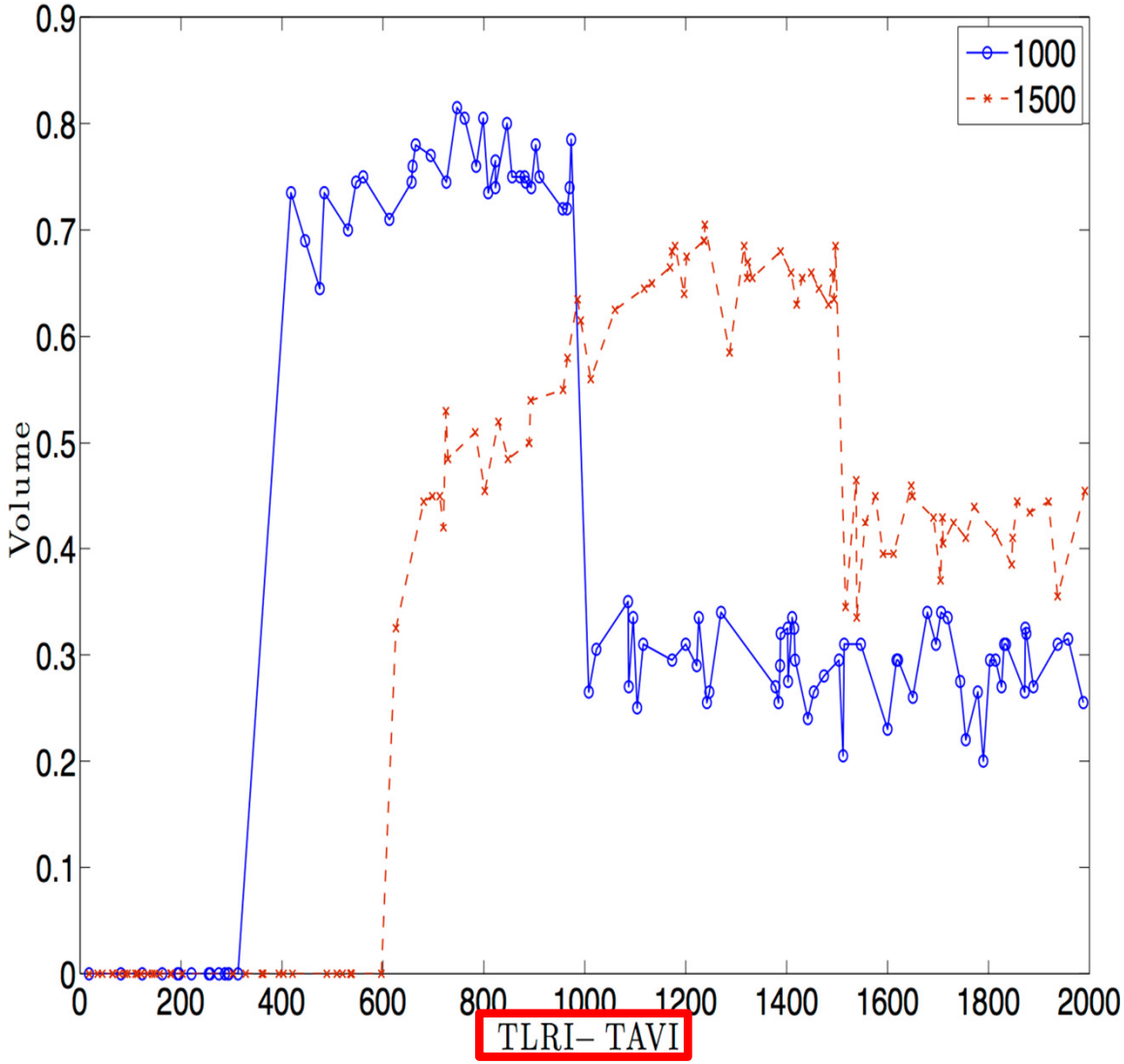
Human heart timed I/O automata model



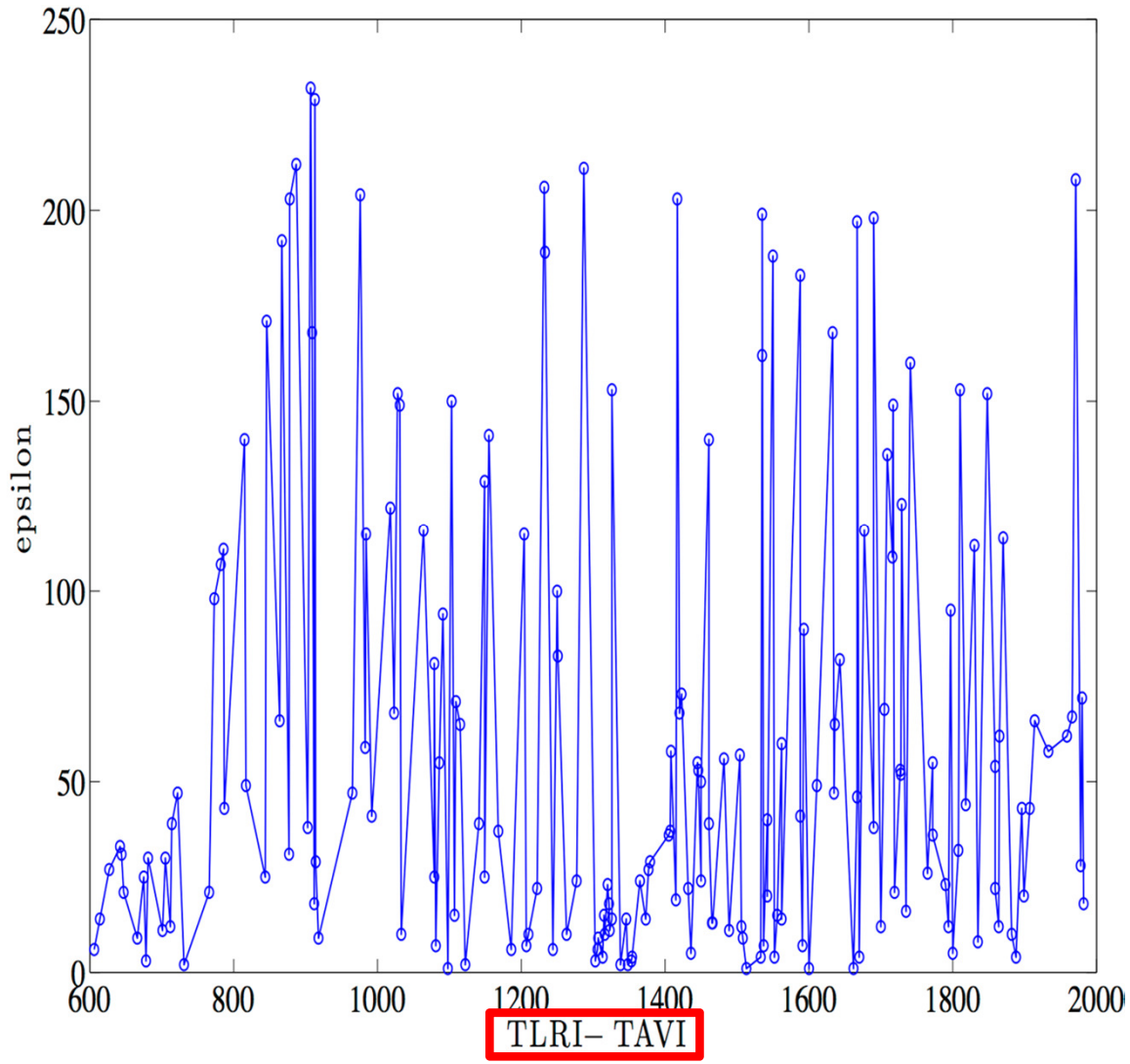




Results: maximal volume objective (PP)



Results: robust objective (PP)

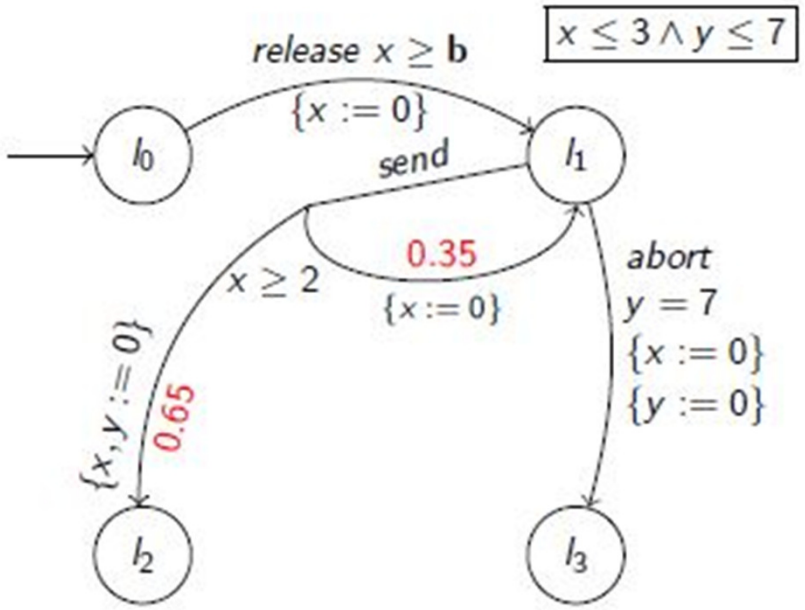


2. Optimal probability timing delays

- Previously, no nondeterminism and no probability in the model considered
- Consider parametric probabilistic timed automata (PPTA),
  - e.g. guards of the form $x \leq b$,
- can we synthesise optimal timing parameters to optimise the reachability probability?
- Semi-algorithm
  - exploration of parametric symbolic states, i.e. location, time zone and parameter valuations
  - forward exploration only gives upper bounds on maximum probability (resp. lower for minimum)
  - but stochastic game abstraction yields the precise solution…
  - expected time challenging
- Implementation in progress

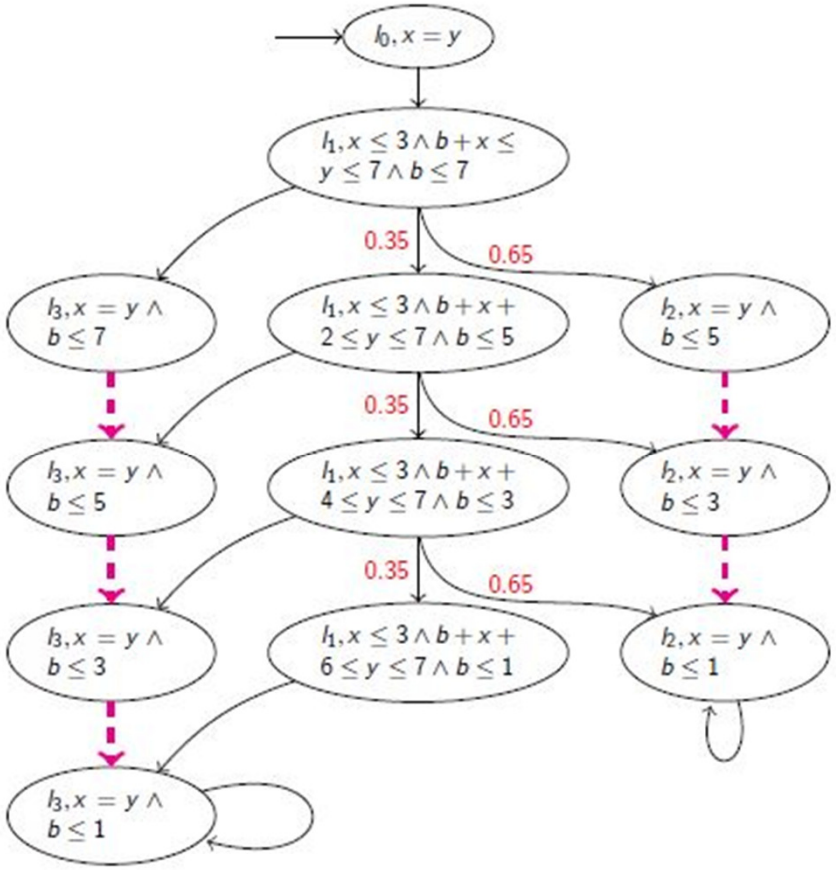
Parameter Synthesis for Probabilistic Timed Automata Using Stochastic Games. Jovanovic and Kwiatkowska. In *Proc. 8th International Workshop on Reachability Problems (RP'14)*, 2014.

Example: parametric PTA



- Consider maximum probability of reaching l_2
  - b = 0, 1: 0.957125
  - b = 2, 3: 0:8775
  - b = 4, 5: 0.65
  - b > 6: 0

Example (MDP abstraction)



max probab of l_2

- b = 0, 1: 0.957125
- b = 2, 3: 0:8775
- b = 4, 5: 0.65
- b > 6: 0

3. Optimal rates

- Motivation: systems and synthetic biology
  - signalling pathways, gene regulation, epidemic models
  - DNA logic gates, DNA walker circuits
  - low molecular counts => stochastic dynamics
  - semantics given by continuous-time Markov chains (CTMCs)

- Uncertain kinetic parameters
  - limited knowledge of rate parameters
  - parameters affect behaviour and functionality of systems
  - NB safety-critical if used in biosensors…

- Can we find rate values so that a reliability property is satisfied?

Precise Parameter Synthesis for Stochastic Biochemical Systems. Ceska et al. In *Proc. 12th Conference on Computational Methods in Systems Biology (CMSB'14)*, 2014.

Optimal rates

- Models: continuous-time Markov chains
  - real-time, exponentially distributed delays
  - extend with rate parameters, bounded parameter space
  - no nondeterminism (add priority and urgency of output)
- CTMCs for biochemical reaction networks
  - state = vector of populations
  - transition rates given by rate parameters using rate functions
  - low degree polynomial functions (mass action kinetics, etc.)
- Properties: Continuous Stochastic Logic
  - time-bounded fragment, branching-time logic
  - probability and reward operators
  - example path formula $\phi = F^{[1000;1000]}\ 15 \leq X \leq 20$
  - Two variants: find rates so that the probability/reward of ϕ meets threshold (say 40%), or is optimised

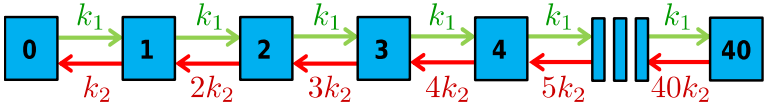
Problem formulation

- Parametric CTMC pCTMC
  - transition rates depend on a set of variables K
  - parametric rate matrix R^K (polynomials with variables in K)
  - describes set of all instantiations C

- Satisfaction function Λ
  - let ϕ be a CSL path formula
  - Λ(p) yields probability of ϕ being satisfied in states s of C
  - analytical computation of Λ is intractable
  - can be discontinuous due to nested probabilistic operators

Example: satisfaction function

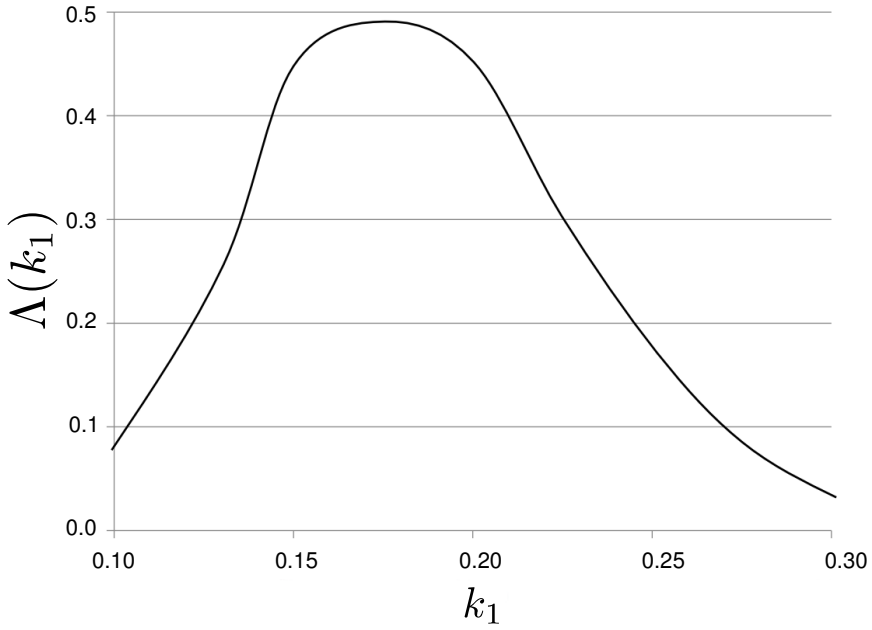
pCTMC + property



$$\phi = \mathrm{F}^{[1000,1000]}(X \geq 15 \wedge X \leq 20)$$

$$k_1 \in [0.1, 0.3] \quad k_2 = 0.01 \quad s_0 = [X]_0 = 15$$

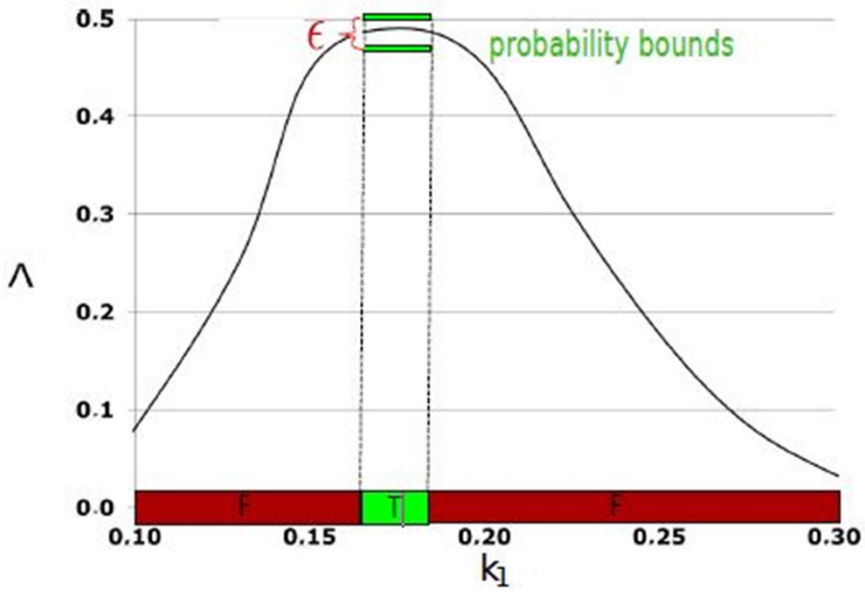
Satisfaction function



Max synthesis problem

For a given $\mathcal{P}$, ϕ and probability tolerance ϵ the problem is finding a partition $\{T, F\}$ of $\mathcal{P}$ and probability bounds $\Lambda^{\perp}$, $\Lambda^{\top}$ such that:

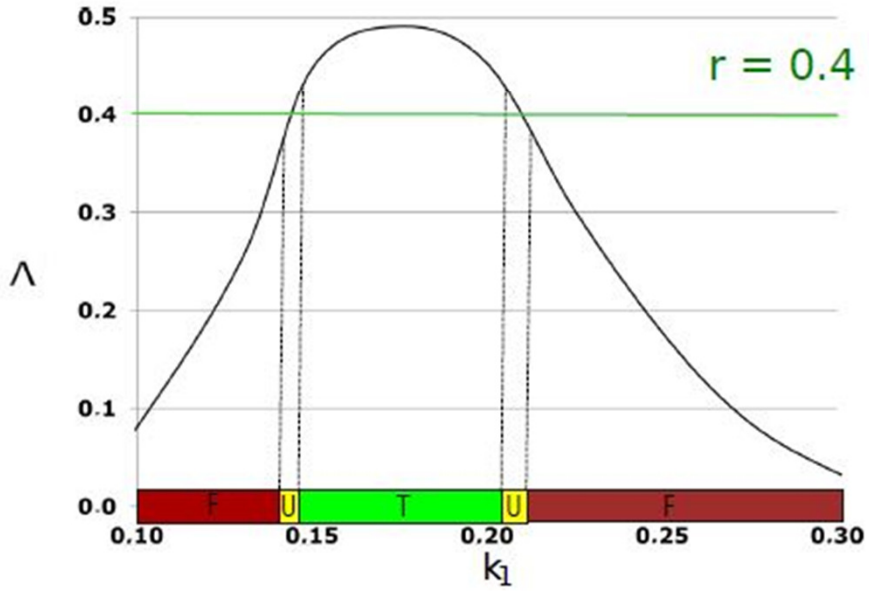
1. $\Lambda^{\perp} - \Lambda^{\top} \leq \epsilon$;
2. $\forall p \in T.\ \Lambda^{\perp} \leq \Lambda(p) \leq \Lambda^{\top}$; and
3. $\exists p \in T.\ \forall p' \in F.\ \Lambda(p) > \Lambda(p')$.



Threshold synthesis

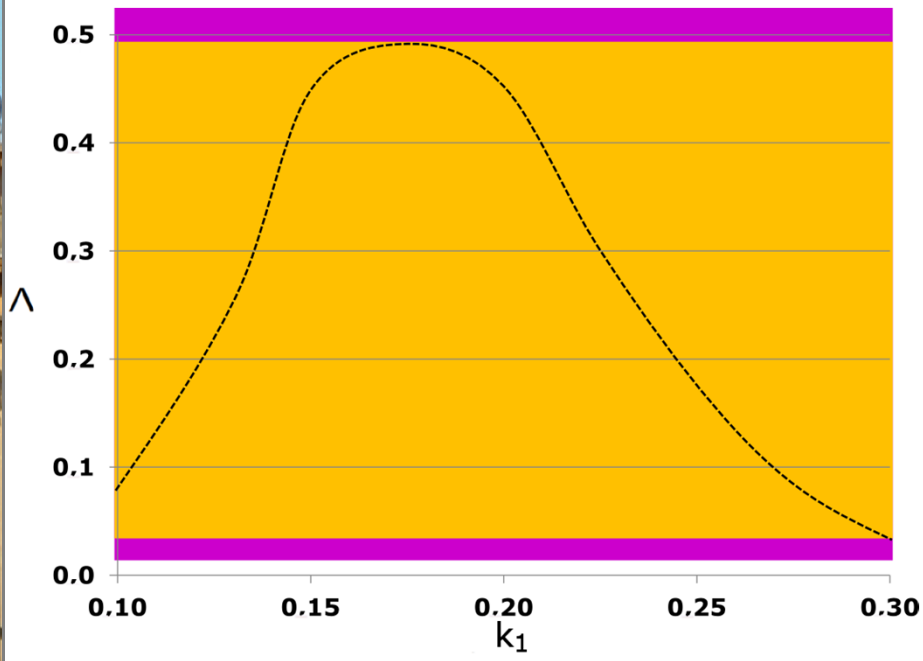
For a given $\mathcal{P}$, ϕ, probability threshold r and volume tolerance ε, the problem is finding a partition $\{T, U, F\}$ of $\mathcal{P}$ such that

1. $\forall p \in T.\ \Lambda(p) \geq r$; and
2. $\forall p \in F.\ \Lambda(p) < r$; and
3. $\mathrm{vol}(U)/\mathrm{vol}(\mathcal{P}) \leq \varepsilon$ ($\mathrm{vol}(A)$ is the volume of A).



Solution approach

1. Method to compute safe approximations to min and max probabilities over a fixed parameter region



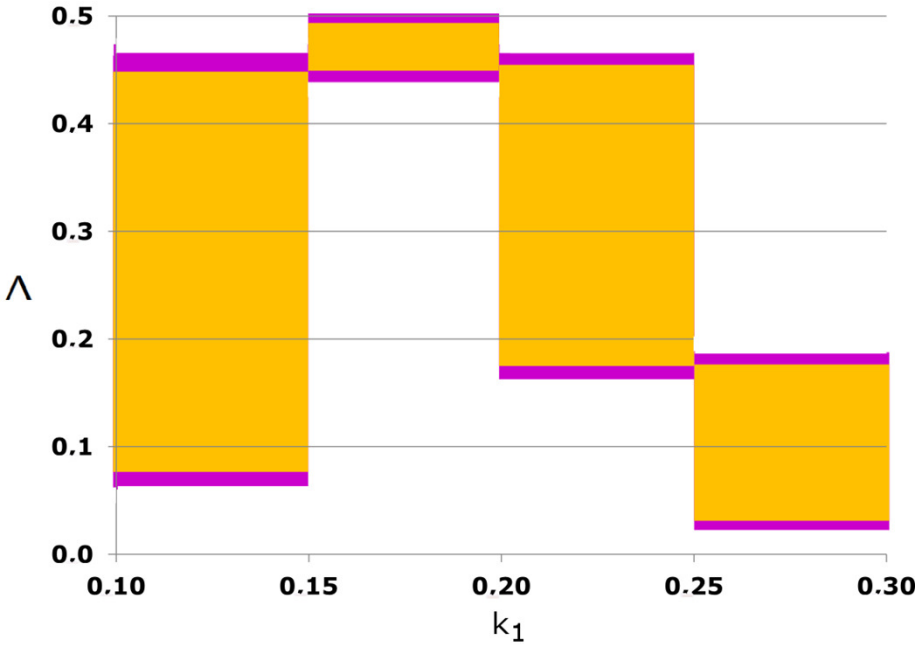
Upper and lower bounds

Safe approximations

Iterative procedure, safe approximations computed for each subregion, same asymptotic complexity as transient analysis

Solution approach

1. Method to compute safe approximations to min and max probabilities over a fixed parameter region
2. Parameter space decomposition, improves accuracy



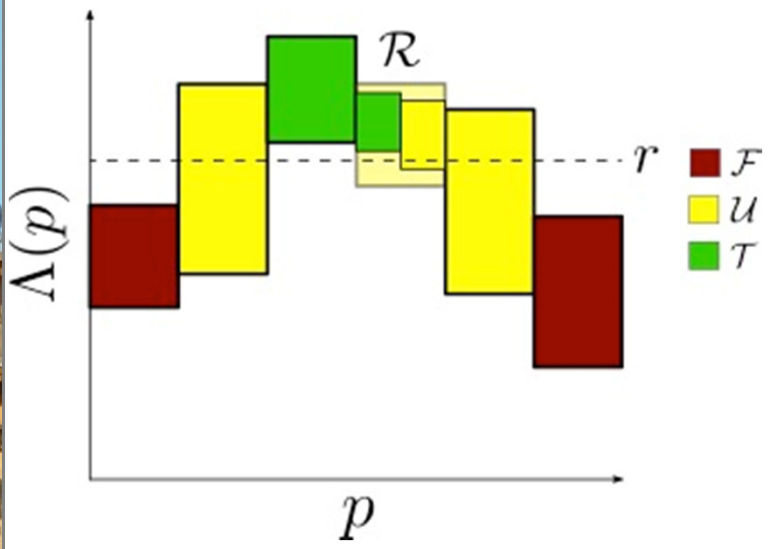
Upper and lower bounds

Safe approximations

Λ is piecewise polynomial function, additional checks for jump discontinuities needed

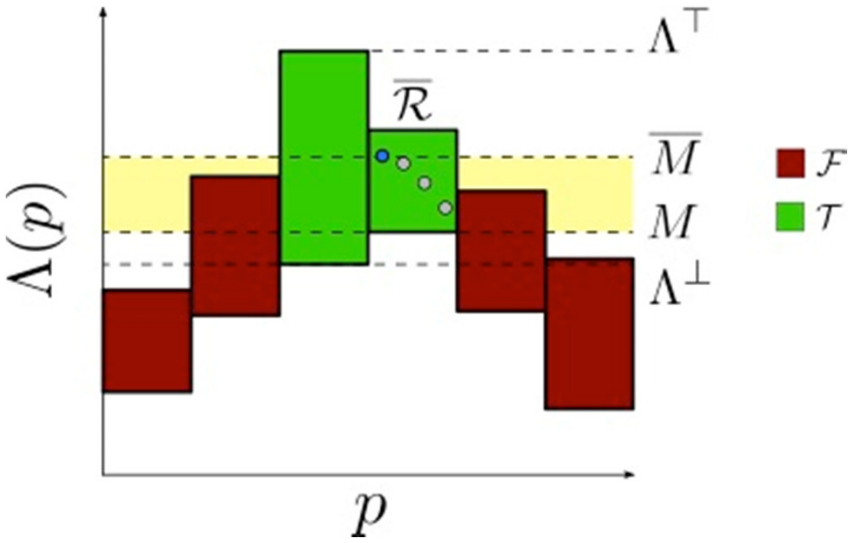
Example: synthesis

Threshold ($\geq r$)



- True if lower bound above r
- False if upper bound below r
- Undecided otherwise (to refine)

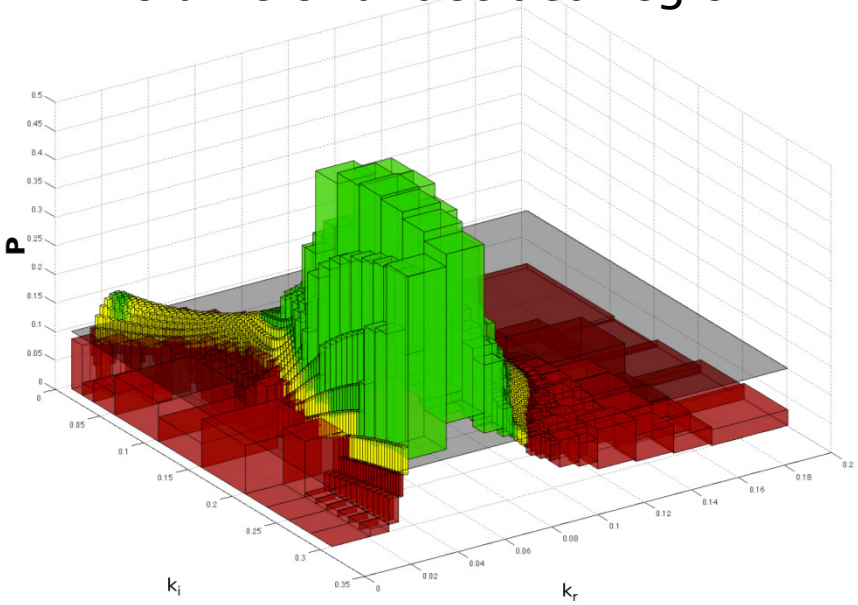
Max

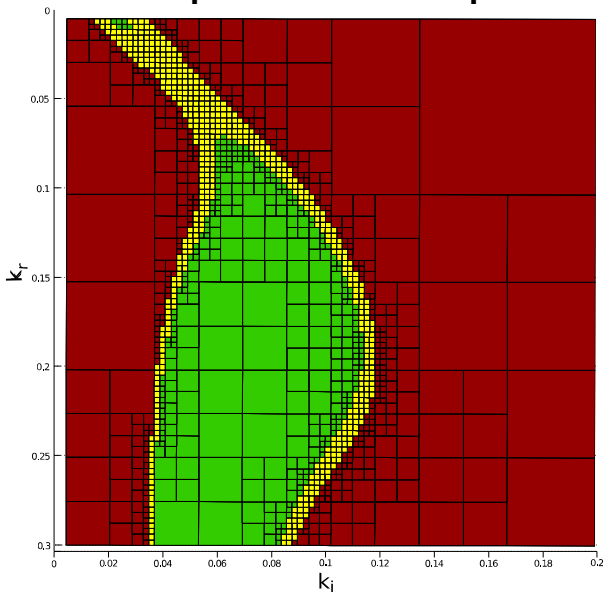


- False if upper bound below under-approximation of max prob M
- True otherwise (to refine)

Epidemic model: threshold synthesis

- probability of property $\geq$ 10%
- volume of undecided region $\leq$ 10% volume of the parameter space





$S + I \xrightarrow{k_i} I + I$ Susceptible → Infected

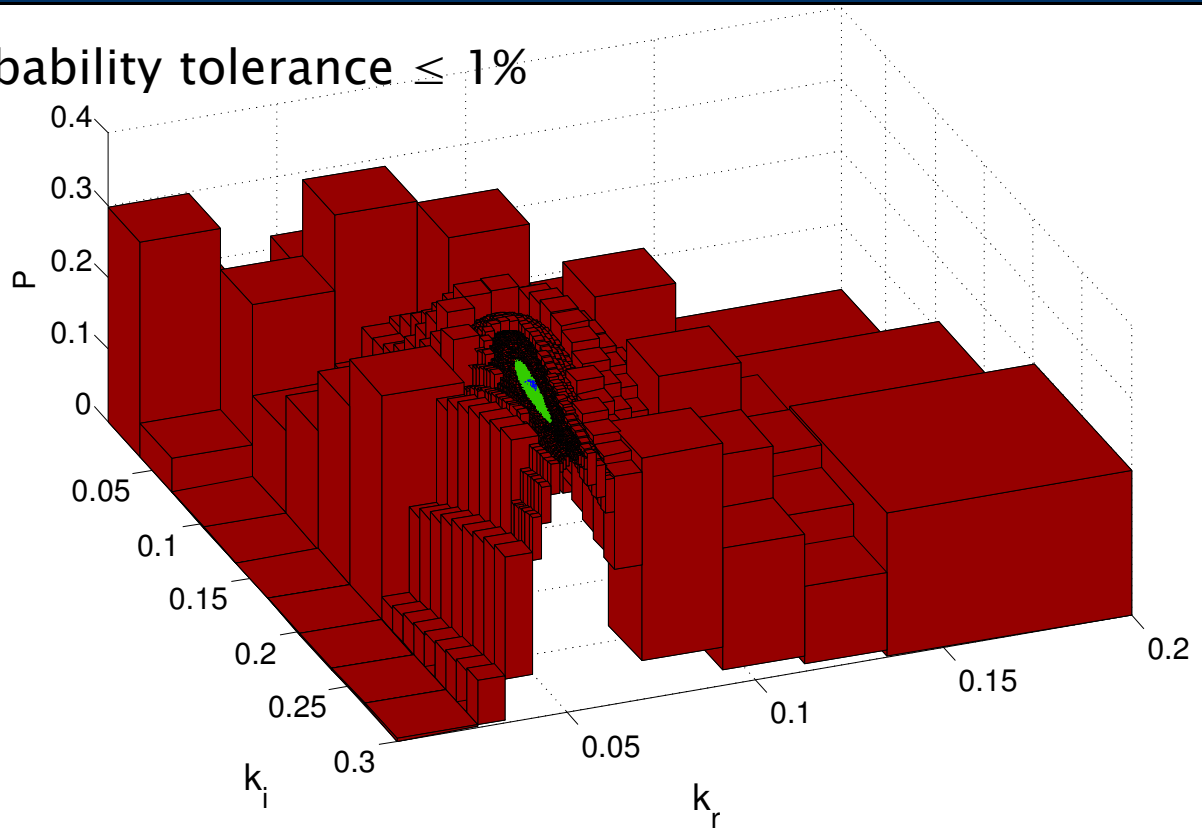
$I \xrightarrow{k_r} R$ Infected → Recovered

k_i k_r uncertain parameters

Property: $\phi = (I > 0)U^{[100,120]}(I = 0)$ (infection lasts at least 100 time units and ends within 120 time units)

Epidemic model: max synthesis

- probability tolerance ≤ 1%



$S + I \xrightarrow{k_i} I + I$ Susceptible → Infected

$I \xrightarrow{k_r} R$ Infected → Recovered

k_i k_r uncertain parameters

Property: $\phi = (I > 0)U^{[100,120]}(I = 0)$ (infection lasts at least 100 time units and ends within 120 time units)

Conclusions

- Formulated and proposed solutions to parameter synthesis problems for probabilistic real-time systems
  - parametric timing delays and rates
  - synthesise constraints or optimal parameters
  - variety of objectives
- Techniques
  - discretisation and integer parameters
  - constraint solving, including parametric symbolic constraints
  - iterative refinement to improve accuracy
  - sampling to improve efficiency
  - but scalability is still the biggest challenge
- Implementation
  - using tool combination involving Z3, python, PRISM

Other work and future directions

- Many challenges remain
  - timed automata models with data
  - hybrid automata models
  - effective model combinations of techniques
  - parallelisation and approximate methods
  - model synthesis from specifications
- More work not covered in this lecture
  - controller synthesis from multiobjective specifications
  - compositional controller synthesis
  - controller synthesis using machine learning
  - code generation
  - new application domains, …
- and more…

Acknowledgements

- My collaborators in this work
- Project funding
  - ERC, EPSRC LSCITS
  - Oxford Martin School, Institute for the Future of Computing

- See also
  - PRISM www.prismmodelchecker.org
  - VERIWARE www.veriware.org