Model checking and strategy synthesis for mobile autonomy: from theory to practice

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Mobile autonomy is here
Software everywhere

- Users expect: **predictability & high integrity** in presence of
  - component failure, environmental uncertainty, ...
  - can be quantified probabilistically

- **Quantitative properties**
  - safety, reliability, performance, ...
  - “the probability of an airbag failing to deploy within 0.02s”

- **Quantitative verification to the rescue**
  - temporal logic specifications
  - quantitative verification
Quantitative verification

• Employ (quantitative) formal models
  – rigorous, unambiguous
  – can be derived or extracted from code
  – can also be used at runtime

• Specify goals/objectives/properties in temporal logic:
  – reliability, energy efficiency, performance, resource usage, ...
  – (reliability) “alert signal will be delivered with high probability in 10ms”, for in-car communication
  – (energy) “maximum expected energy consumption in 1 hr is at most 10mA”, for an autonomous robot

• Focus on automated, tool-supported methodologies
  – model-based design
  – automated verification via model checking
  – controller synthesis from (temporal logic) specifications
  – NB employed in control, cf Murray’s work
Quantitative/probabilistic verification

Automatic verification (aka model checking) of quantitative properties of probabilistic system models

System requirements

Probabilistic model
  e.g. Markov chain

Probabilistic model checker
  e.g. PRISM

Result

Quantitative results

Counter-example

System

Probabilistic temporal logic specification
  e.g. PCTL, CSL, LTL

\[ P_{<0.01} \left[ F \leq \text{fail} \right] \]
Quantitative/probabilistic verification

- Property specifications based on temporal logic
  - PCTL, CSL, probabilistic LTL, PCTL*, ...

- Simple examples:
  - $P_{\leq 0.01} \left[ F \text{ “fail” } \right]$ – “the probability of airbag failure is at most 0.01”
  - $S_{>0.999} \left[ \text{ “up” } \right]$ – “long-run probability of availability is $>0.999$”

- Usually focus on quantitative (numerical) properties:
  - $P_{=?} \left[ F \text{ “crash” } \right]$  
    “what is the probability of a crash occurring?”
  - then analyse trends in quantitative properties as system parameters vary
Historical perspective

• First algorithms proposed in 1980s
  – algorithms [Vardi, Courcoubetis, Yannakakis, …]
  – [Hansson, Jonsson, de Alfaro] & first implementations

• 2000: general purpose tools released
  – PRISM: efficient extensions of symbolic model checking
    [Kwiatkowska, Norman, Parker, …]
  – ETMCC: model checking for continuous-time Markov chains [Baier, Hermanns, Haverkort, Katoen, …]

• Now mature area, of industrial relevance
  – successfully used by non-experts for many application domains, but full automation and good tool support essential
    • distributed algorithms, communication protocols, security protocols, biological systems, quantum cryptography, planning, …
  – genuine flaws found and corrected in real-world systems
  – www.prismmodelchecker.org
But which modelling abstraction?

- Several probabilistic models supported…
- Markov chains (DTMCs and CTMCs)
  - discrete states + discrete or exponential probability
  - for: component failures, unreliable communication media, …
- Markov decision processes (MDPs)
  - probability + decisions (nondeterministic choices)
  - for: distributed coordination, motion planning in robotics…
- Probabilistic timed automata (PTAs)
  - probability + decisions + real-time passage
  - for: wireless comm. protocols, embedded control systems, …
- Towards stochastic hybrid systems
  - probability + decisions + continuous flows
  - for: control of physical processes, motion in space,…
The challenge of mobile autonomy

- **Autonomous systems**
  - are reactive, continuously interact with their environment
    - including other components or human users, adversarial
  - have goals/objectives
    - often quantitative, may conflict
  - take decisions based on current state and external events
- **Natural to adopt a game–theoretic view**
  - need to account for the uncontrollable behaviour of components, possibly with differing/opposing goals
  - in addition to controllable events
- **Many occurrences in practice**
  - e.g. decision making in economics, power distribution networks, motion planning, security, distributed consensus, energy management, sensor network co-ordination, semi-autonomous driving...
This lecture…

• Introduce stochastic multi-player games (SMGs)
  – argue that games are an appropriate modelling abstraction for competitive behaviour, in adversarial environments
  – stochasticity to model e.g. failure, sensor uncertainty

• Property specification: rPATL
  – single-objective properties
  – verification
  – strategy synthesis

• Extensions
  – multiobjective properties, Pareto sets
  – compositional strategy synthesis

• Tool support: PRISM-games 2.0

• Case studies

What makes a game?

- Players with moves (turn-based or concurrent)
- Strategy for each player
  - plans for how to choose moves, based on information available
- Value (or payoff) for each player
- Winning
  - corresponds to optimising the value no matter how the others play the game
- Main question: is there a winning strategy?
Playing games with Google car…

Are you going? Or should I go?

What if I point a lot and flail my arms around?

You go first.

This is confusing.

Wait, maybe you should go.

Let's just sit here and reflect.

– http://theoatmeal.com/blog/google_self_driving_car
Stochastic multi-player games (SMGs)

- **A stochastic game involves**
  - multiple players (competitive or collaborative behaviour)
  - nondeterminism (decisions, control, environment)
  - probability (failures, noisy sensors, randomisation)

- **Here consider only**
  - turn-based, discrete time, zero sum, complete observation
  - timed/continuous extensions exist, but tool support lacking

- **Many applications**
  - autonomous traffic (risk averse vs risk taking)
  - distributed coordination (selfish agents vs unselfish)
  - controller synthesis (system vs. environment)
  - security (defender vs. attacker)
Stochastic multi-player games

- **Stochastic multi-player game (SMGs)**
  - multiple players + nondeterminism + probability
  - generalisation of MDPs: each state controlled by unique player

- **A (turn-based) SMG is a tuple** $(\Pi, S, \langle S_i \rangle_{i \in \Pi}, A, \Delta, L)$:
  - $\Pi$ is a set of $n$ players
  - $S$ is a (finite) set of states
  - $\langle S_i \rangle_{i \in \Pi}$ is a partition of $S$
  - $A$ is a set of action labels
  - $\Delta : S \times A \rightarrow \text{Dist}(S)$ is a (partial) transition probability function
  - $L : S \rightarrow 2^{\text{AP}}$ is a labelling with atomic propositions from $\text{AP}$

- **Notation:**
  - $A(s)$ denotes available actions in state $s$
Rewards

- Annotate SMGs with rewards (or costs)
  - real-valued quantities assigned to states (and/or transitions)

- Wide range of possible uses:
  - elapsed time, power consumption, number of messages successfully delivered, net profit, ...

- We work with:
  - state rewards: $r : S \rightarrow \mathbb{R}_{\geq 0}$

- Form basis for a variety of quantitative objectives
  - expected cumulative (total) reward (denoted C)
  - mean-payoff (limit-average) reward (denoted S)
  - ratio reward
  - (can also consider discounted reward)
Paths, strategies + probabilities

• A path is an (infinite) sequence of connected states in SMG
  – i.e. $s_0a_0s_1a_1...$ such that $a_i \in A(s_i)$ and $\Delta(s_i,a_i)(s_{i+1}) > 0$ for all $i$
  – represents a system execution (i.e. one possible behaviour)
  – to reason formally, need a probability space over paths

• A strategy for player $i \in \Pi$ resolves choices in $S_i$ states
  – based on history of execution so far
  – i.e. a function $\sigma_i : (SA)^*S_i \rightarrow \text{Dist}(A)$
  – $\Sigma_i$ denotes the set of all strategies for player $i$
  – deterministic if $\sigma_i$ always gives a Dirac distribution
  – memoryless if $\sigma_i(s_0a_0...s_k)$ depends only on $s_k$
  – also finite–memory, infinite memory, ...
  – history based or explicit memory representation

• A strategy profile is tuple $\sigma=(\sigma_1,...,\sigma_n)$
  – combining strategies for all $n$ players
Paths, strategies + probabilities...

• For a strategy profile $\sigma$:
  – the game’s behaviour is fully probabilistic
  – essentially an (infinite-state) Markov chain
  – yields a probability measure $\Pr_s^\sigma$
    over set of all paths $\text{Path}_s$ from $s$

• Allows us to reason about the probability of events
  – under a specific strategy profile $\sigma$
  – e.g. any ($\omega$-)regular property over states/actions

• Also allows us to define expectation of random variables
  – i.e. measurable functions $X : \text{Path}_s \to \mathbb{R}_{\geq 0}$
  – $E_s^\sigma[X] = \int_{\text{Path}_s} X \, d\Pr_s^\sigma$
  – used to define expected costs/rewards...
Property specification: rPATL

- **Temporal logic rPATL:**
  - reward probabilistic alternating temporal logic

- **CTL, extended with:**
  - coalition operator $\langle\langle C \rangle\rangle$ of ATL (Alternating Temporal Logic)
  - probabilistic operator $P$ of PCTL, where $P_{\bowtie_q}[\psi]$ means “the probability of ensuring $\psi$ satisfies $\bowtie q$”
  - reward operator $R$ of PRISM, where $R_{\bowtie_q}[\rho]$ means “the expected value of $\rho$ satisfies $\bowtie q$”

- **Example:**
  - $\langle\langle\{1,2\}\rangle\rangle P_{<0.01}[F_{\leq 10} \text{ error }]$
  - “players 1 and 2 have a strategy to ensure that the probability of an error occurring within 10 steps is less than 0.1, regardless of the strategies of other players”
rPATL properties

• Syntax:

\[ \phi ::= \langle\langle C \rangle\rangle P_{\bowtie q}[\psi] | \langle\langle C \rangle\rangle R_{\bowtie q}^r[\rho] | \langle\langle C \rangle\rangle R_{\bowtie q}^{r/c}[\rho] \]

\[ \psi ::= F a \quad \text{“reachability”} \]

\[ \rho ::= C | S \quad \text{“longrun average”} \]

• where:

- \( a \in AP \) is an atomic proposition, \( C \subseteq \Pi \) is a coalition of players,
  \( \bowtie \in \{\leq, <, >, \geq\} \), \( q \in \mathbb{R}_{\geq 0} \), \( r \) and \( c \) are reward structures

• \( \langle\langle C \rangle\rangle P_{\geq 1}[F \text{ “end”}] \)
  - “players in coalition C have a collective strategy to ensure that the game reaches an “end”–state almost surely, regardless of the strategies of other players”
rPATL reward properties

- **Syntax:**

\[ \phi ::= \langle\langle C\rangle\rangle P_{\bowtie q}[\psi] \mid \langle\langle C\rangle\rangle R_{\bowtie q}[\rho] \mid \langle\langle C\rangle\rangle R_{r/c\bowtie q}[\rho] \]

\[ \psi ::= F a \]

\[ \rho ::= C \mid S \]

- \( \langle\langle C\rangle\rangle R_{\text{fuel} < q}[C] \)
  - “players in coalition C have a strategy to ensure that the expected total fuel consumption is less than q, regardless of the strategies of other players”

- \( \langle\langle C\rangle\rangle R_{\text{fuel/time} \leq q}[S] \)
  - “players in coalition C have a strategy to ensure that the expected longrun fuel consumption per time unit is at most q, regardless of the strategies of other players”
• Semantics for most operators is standard
• Just focus on P and R operators…
  – use reduction to a stochastic 2–player game

• Coalition game $G_C$ for SMG $G$ and coalition $C \subseteq \Pi$
  – 2–player SMG where $C$ and $\Pi \setminus C$ collapse to players 1 and 2

• $\langle\langle C \rangle \rangle P \bowtie q [\psi]$ is true in state $s$ of $G$ iff:
  – in coalition game $G_C$:
  – $\exists \sigma_1 \in \Sigma_1$ such that $\forall \sigma_2 \in \Sigma_2$. $Pr_s^{\sigma_1,\sigma_2}(\psi) \bowtie q$

• Semantics for R operator defined similarly…
Examples

\[ \langle \langle \circ \rangle \rangle P_{\geq \frac{1}{4}} [ F \, \checkmark ] \]

true in initial state

\[ \langle \langle \circ \rangle \rangle P_{\geq \frac{1}{3}} [ F \, \checkmark ] \]

\[ \langle \langle \circ \rangle \rangle P_{\geq \frac{1}{3}} [ F \, \checkmark ] \]
Examples

\[ \langle\langle \cdot \rangle\rangle_{P_{\geq \frac{1}{4}}} [ F \checkmark ] \]
\text{true in initial state}

\[ \langle\langle \cdot \rangle\rangle_{P_{\geq \frac{1}{3}}} [ F \checkmark ] \]
\text{false in initial state}

\[ \langle\langle \cdot, \square \rangle\rangle_{P_{\geq \frac{1}{3}}} [ F \checkmark ] \]
Examples

\[\langle\langle \bigcirc \rangle\rangle \geq \frac{1}{4}[F \checkmark]\]
true in initial state

\[\langle\langle \bigcirc \rangle\rangle \geq \frac{1}{3}[F \checkmark]\]
false in initial state

\[\langle\langle \bigcirc, \square \rangle\rangle \geq \frac{1}{3}[F \checkmark]\]
true in initial state
Verification and strategy synthesis

- **The verification problem is:**
  - Given a game $G$ and rPATL property $\phi$, does $G$ satisfy $\phi$?

- **e.g.** $\langle\langle C\rangle\rangle P \bowtie q[\psi]$ is true in state $s$ of $G$ iff:
  - in coalition game $G_C$:
    - $\exists \sigma_1 \in \Sigma_1$ such that $\forall \sigma_2 \in \Sigma_2$. $Pr_{s,\sigma_1,\sigma_2}(\psi) \bowtie q$

- **The synthesis problem is:**
  - Given an SMG $G$ and a coalition property $\phi$, find, if it exists, a coalition strategy $\sigma$ that is a witness to $G$ satisfying $\phi$

- **Reduce to computing optimal values and winning strategies in 2–player games**
  - (epsilon–optimal) strategies can be typically extracted from optimal values in linear time (under restrictions)
Model checking for rPATL

• Basic algorithm: as for any branching-time temporal logic

• Main task: checking P and R operators
  – reduction to solution of stochastic 2-player game $G_C$
  – e.g. $\langle\langle C \rangle\rangle P_{\geq q} [\psi] \iff \sup_{\sigma_1 \in \Sigma_1} \inf_{\sigma_2 \in \Sigma_2} \Pr_{s, \sigma_1, \sigma_2} (\psi) \geq q$
  – complexity: NP $\cap$ coNP (this fragment,)
  – no P algorithm known, compare to, e.g., P for Markov decision processes

• Quantitative (numerical) properties:
  – best/worst-case values

• e.g. $\langle\langle C \rangle\rangle P_{\max=?} [\psi] = \sup_{\sigma_1 \in \Sigma_1} \inf_{\sigma_2 \in \Sigma_2} \Pr_{s, \sigma_1, \sigma_2} (\psi)$

• In practice:
  – employ value iteration
  – up to a desired level of convergence
Probabilities for P operator

- E.g. $\langle \langle C \rangle \rangle P_{\geq q}[ F a ]$: max/min reachability probabilities
  - compute $\sup_{\sigma_1 \in \Sigma_1} \inf_{\sigma_2 \in \Sigma_2} \Pr_{s_{\sigma_1, \sigma_2}} (F a)$ for all states $s$
  - deterministic memoryless strategies suffice
- Value is:
  - 1 if $s \in \text{Sat}(a)$, and otherwise least fixed point of:
    $$f(s) = \begin{cases} 
    \max_{a \in A(s)} \left( \sum_{s' \in S} \Delta(s, a)(s') \cdot f(s') \right) & \text{if } s \in S_1 \\
    \min_{a \in A(s)} \left( \sum_{s' \in S} \Delta(s, a)(s') \cdot f(s') \right) & \text{if } s \in S_2 
    \end{cases}$$
- Computation:
  - start from zero, propagate probabilities backwards
  - guaranteed to converge
Example

rPATL: \(\langle\bigcirc, \blacksquare\rangle P_{\geq \frac{1}{3}} [F \, \checkmark]\)

Player 1: \(\bigcirc, \blacksquare\) Player 2: \(\Diamond\)

Compute: \(\sup_{\sigma_1 \in \Sigma_1} \inf_{\sigma_2 \in \Sigma_2} Pr_{s^{\sigma_1, \sigma_2}}(F \, \checkmark)\)
Strategy synthesis for rPATL

• The verification problem is
  – Given a game G and rPATL property φ, does G satisfy φ?
• e.g. $\langle\langle C \rangle\rangle P_{\bowtie q} [\psi]$ is true in state $s$ of $G$ iff:
  – in coalition game $G_C$:
  – $\exists \sigma_1 \in \Sigma_1$ such that $\forall \sigma_2 \in \Sigma_2 . \Pr_{s,\sigma_1,\sigma_2} (\psi) \bowtie q$

• The synthesis problem is
  – Given a game G and rPATL property φ, find, if it exists, a coalition strategy $\sigma_1$ that is a witness to the satisfaction of φ

• In other words
  – find a controller working under all environment conditions, including adversarial
  – memoryless deterministic strategies suffice (this fragment)
Multi-objective properties

- Extension of rPATL: **conjunctions** of objectives (for stopping games), i.e. multidimensional
  - expected total rewards, mean-payoffs or ratios
  - almost sure mean-payoffs/ratios

- Explore trade-offs
  - e.g. between performance and resource usage

- Example
  - “coalition C can guarantee that the expected longrun average fuel consumption and profit are simultaneously at least \( v_1 \) and \( v_2 \), respectively, no matter what the other agents do”
    \[ \langle \langle C \rangle \rangle (R_{\text{fuel}} \geq v_1 [S] \& R_{\text{profit}} \geq v_2 [S]) \]

- NB Boolean combinations may be needed for implication
  \[ \langle \langle C \rangle \rangle (R_{\text{fuel/time}} \geq v_1 [S] \Rightarrow R_{\text{profit}} \geq v_2 [S]) \]
• Consider the simpler scenario of MDPs

• Pareto optimum for multiple objectives
  – probability of reaching $D$ is greater than 0.2 \textbf{and}
  – probability of reaching $E$ is greater than 0.6
Computation of Pareto sets

• Multi-objective strategy synthesis
  – epsilon-optimal strategies, randomised
  – value iteration over polytopic sets
  – stochastic memory update representation

• Pareto sets
  – optimal achievable trade-offs between objectives

• Visualisation of high-dimensional Pareto sets
  – projection
  – slicing
Multidimensional Pareto set

Pareto set approximation for a mixed multi-objective property
Compositional strategy synthesis

- **Components**
  - reduce design complexity, increase reliability via redundancy
  - improve scalability of analysis, avoid product state space

- **Assume–guarantee synthesis:**
  - need a strategy for the full system satisfying a **global property**
  - synthesise one strategy per component, for **local properties**
  - use **assume–guarantee rules** to compose local strategies

- **Example:** local strategies for $G_1 \models \phi^A$ and $G_2 \models \phi^A \Rightarrow \phi^B$ compose to a **global strategy** for $G_1 \parallel G_2 \models \phi^B$

- **Need to extend synthesis methods:**
  - **multi–objective properties** to use in local and global properties
  - admit also long–run properties (e.g. ratios of rewards)
Compositional strategy synthesis

- Based on **assume-guarantee contracts** over component interfaces
- Synthesise **local** strategies for components, then compose into a **global** strategy using assume-guarantee rules
- Under-approximation of Pareto sets
Tool support: PRISM-games 2.0

- Model checker for stochastic games
  - integrated into PRISM model checker
  - using new explicit-state model checking engine
- SMGs added to PRISM modelling language
  - guarded command language, based on reactive modules
  - finite data types, parallel composition, proc. algebra op.s, …
- rPATL added to PRISM property specification language
  - implemented value iteration based model checking
- Supports strategy synthesis
  - single and multiple objectives, Pareto curve
  - total expected reward, longrun average, ratio rewards
  - compositional strategy synthesis
- Available now:
  - http://www.prismmodelchecker.org/games/
Case studies

• Evaluated on several case studies:
  – team formation protocol [CLIMA’11]
  – futures market investor model [McIver & Morgan]
  – collective decision making for sensor networks [TACAS’12]
  – energy management in microgrids [TACAS’12]
  – reputation protocol for user-centric networks [SR’13]
  – DNS bandwidth amplification attack [Deshpande et al]
  – self-adaptive software architectures [Camara, Garlan et al]
  – attack–defence scenarios in RFID goods man. [Aslanyan et al]
• Case studies using PRISM-games 2.0 functionality:
  – autonomous urban driving (multi-objective) [QEST’13]
  – UAV path planning with operator (multi-objective) [ICCPS’15]
  – aircraft electric power control (compositional) [TACAS’15]
  – temperature control (compositional) [Wiltsche PhD]

Case study: Energy management

• Energy management protocol for Microgrid
  – Microgrid: local energy management
  – randomised demand management protocol [Hildmann/Saffre'11]
  – probability: randomisation, demand model, …

• Existing analysis
  – simulation-based
  – assumes all clients are unselfish

• Our analysis
  – stochastic multi-player game
  – clients can cheat (and cooperate)
  – exposes protocol weakness
  – propose/verify simple fix

Automatic Verification of Competitive Stochastic Systems, Chen et al., In Proc TACAS 2012
Case study: Autonomous urban driving

- **Inspired by DARPA challenge**
  - represent map data as a stochastic game, with environment *active*, able to select hazards
  - express goals as *conjunctions* of probabilistic and reward properties
  - e.g. “maximise probability of avoiding hazards and minimise time to reach destination”

- **Solution (PRISM–games 2.0)**
  - synthesise a *probabilistic* strategy to achieve the multi-objective goal
  - enable the exploration of *trade-offs* between subgoals
  - applied to synthesise driving strategies for English villages

*Synthesis for Multi–Objective Stochastic Games: An Application to Autonomous Urban Driving*, Chen et al., In *Proc QEST 2013*
Case study: UAV path planning

- Human operator
  - sensor tasks
  - high-level commands for piloting

- UAV autonomy
  - low-level piloting function

- Quantitative mission objectives
  - road network surveillance with the minimal time, fuel, or restricted operating zone visits

- Analysis of trade-offs
  - consider operator fatigue and workload
  - multi-objective, MDP and SMG models

Case study: Aircraft power distribution

- Consider Honeywell high-voltage AC (HVAC) subsystem
  - power routed from generators to buses through switches
  - represent as a stochastic game, modelling competition for buses, with stochasticity used to model failures
  - specify control objectives in LTL using longrun average
    - e.g. “maximise uptime of the buses and minimise failure rate”

- Solution (PRISM-games 2.0)
  - compositional strategy synthesis
  - enable the exploration of trade-offs between uptime of buses and failure rate

Compositional Controller Synthesis for Stochastic Games, Basset et al., In Proc CONCUR 2014
Conclusion

• Summary
  – games can model a wide range of competitive and cooperative goal-driven scenarios relevant for mobile autonomy
  – variety of quantitative objectives
  – multi-objective properties
  – compositional synthesis via assume-guarantee rules
  – implementation: explicit engine, Parma polyhedra library, value iteration
  – high complexity, performance sluggish

• Future work
  – mobility?
  – consider social aspects?
  – allow partial observability?
  – combine with Nash equilibria?

• Beyond games…
• Hybrid model–based framework
  – timed automata model for pacemaker software
  – hybrid heart models in Simulink, adopt synthetic ECG model (non-linear ODE)

• Properties
  – (basic safety) maintain 60–100 beats per minute
  – (advanced) detailed analysis energy usage, plotted against timing parameters of the pacemaker
  – parameter synthesis: find values for timing delays that optimise energy usage

Synthesising robust and optimal parameters for cardiac pacemakers using symbolic and evolutionary computation techniques. Kwiatkowska, Mereacre, Paoletti and Patane, HSB’16
DNA computation

- Cardelli’s DNA transducer gate
  - inputs/outputs single strands
  - two transducers connected
- PRISM identifies a bug: 5-step trace to a “bad” deadlock state
  - previously found manually [Cardelli’10]
  - detection now fully automated
- Bug is easily fixed
  - (and verified)

Counterexample:
(1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,1,0,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,0,1,0,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,0,0,1,0,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,0,0,0,1,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,0,0,0,0,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,0,0,0,0,0,1,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
(0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

DNA origami tiles

- DNA origami tiles: molecular breadboard [Turberfield lab]

Aim to understand how to control the folding pathways
- formulate an abstract Markov chain model
- obtain model predictions using Gillespie simulation
- perform a range of experiments, consistent with predictions

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• See also
  – VERIWARE www.veriware.org
  – PRISM www.prismmodelchecker.org