Model Checking of Probabilistic Systems

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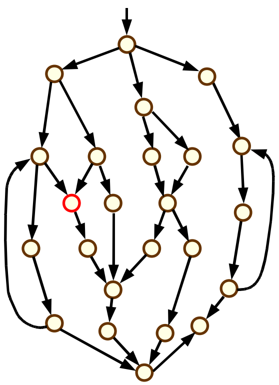
Overview

- Probabilistic model checking
- Probabilistic models
  - DTMCs, CTMCs, MDPs, PTAs, modelling formalisms
- Property specifications
  - PCTL, quantitative results, costs & rewards
- Implementation/tool support
- Current research directions
  - symmetry reduction, compositionality, abstraction/refinement

Verification through model checking

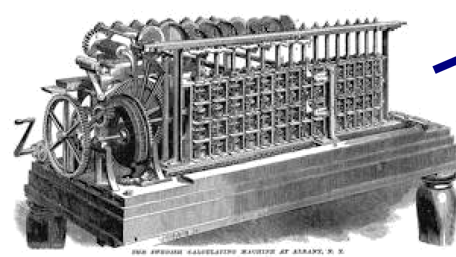
Finite-state model



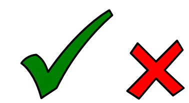
¬EF error

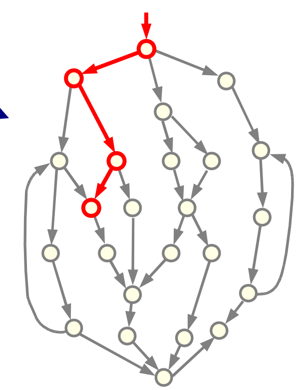
Temporal logic specification

Model checker



Result

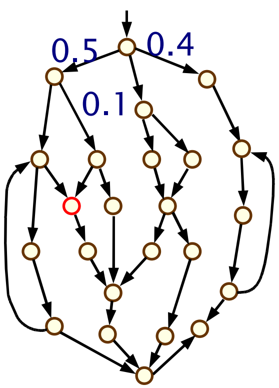




Error trace

Probabilistic model checking

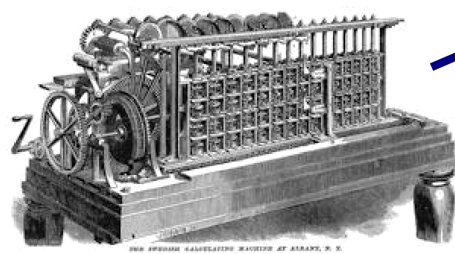
Finite-state probabilistic model
e.g. Markov chain/process



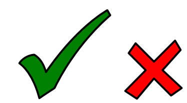
$\mathrm{P}_{<0.01}$ [F error]

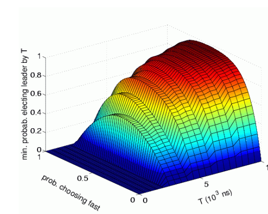
Probabilistic temporal logic specification
e.g. PCTL

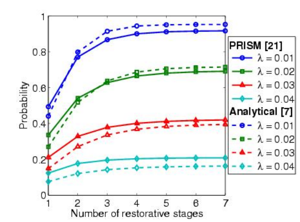
Probabilistic model checker
e.g. PRISM

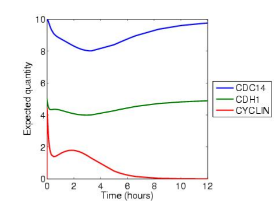


Result



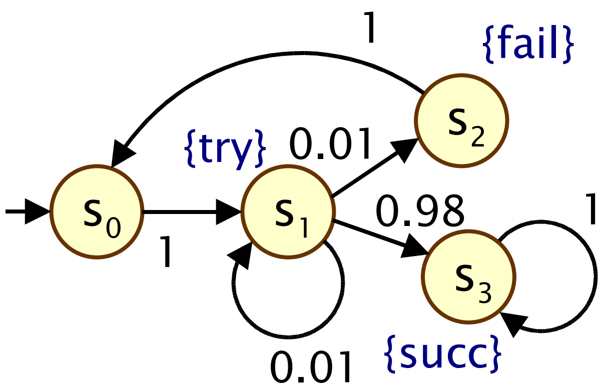






Quantitative results

Discrete-time Markov chains (DTMCs)



Features:

- discrete state space
- discrete time-steps
- discrete probability distributions

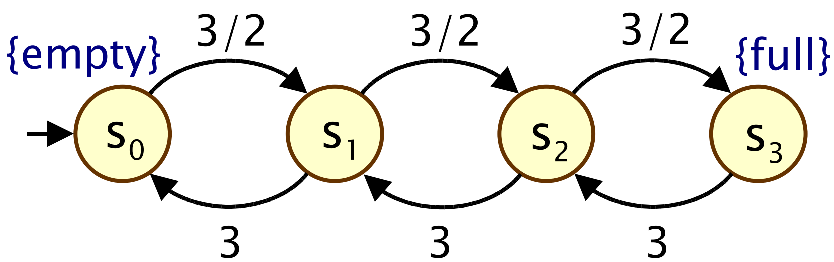
Well suited to modelling:

- randomised algorithms/protocols
- systems with component failures
- assumptions: synchronous (lock-step) parallel composition of components

PRISM case studies:

- Bluetooth device discovery
- probabilistic contract signing
- leader election/self-stabilisation algorithms
- NAND multiplexing
- dynamic power management schemes

Continuous-time Markov chains (CTMCs)



Features:

- discrete state space
- continuous-time (exponentially distributed transition delays)

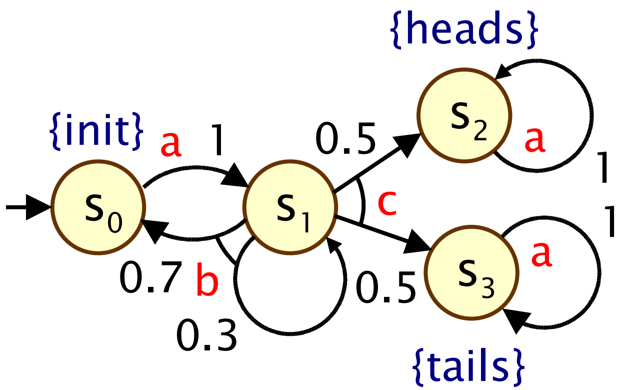
Well suited to modelling:

- component lifetimes, e.g. network cluster
- inter-arrival times, e.g. queueing systems
- biochemical reaction rates, ...

PRISM case studies:

- Dynamic power management schemes
- Queueing/manufacturing systems
- Workstation clusters
- Groupware systems (thinkteam)
- Biological signalling pathways (FGF, Eukaryotes, ...)

Markov decision processes (MDPs)



Features:

- discrete state space, time-steps
- probability and nondeterminism
- nondeterministic choice between multiple probability distributions

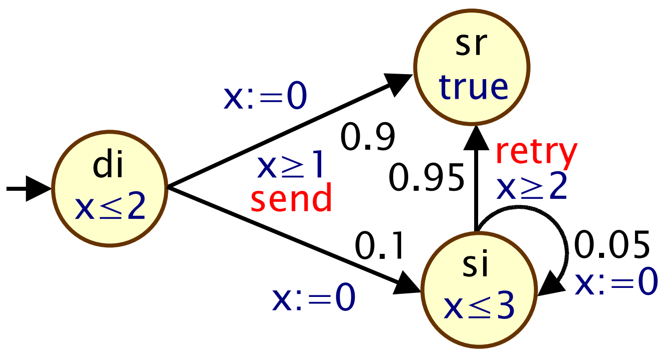
Well suited to modelling:

- asynchronous parallel composition of probabilistic components, e.g. randomised distributed algorithms
- unknown environments, e.g. probabilistic security protocols
- underspecification, e.g. probabilistic communication protocol designed for a range of possible message propagation delays

PRISM case studies:

- Randomised algorithms for self-stabilisation, leader election, consensus, ...
- Firewire/Zeroconf/ZigBee/ CSMA/CD, CSMA/CA
- Probabilistic security protocols for anonymity, contract signing, fair exchange, pin cracking, ...
- Dynamic voltage scaling

Probabilistic timed automata (PTAs)



Features:

- probability + nondeterminism + real-time
- real-valued clocks (extension of timed automata with discrete probabilistic choice)

Well suited to modelling:

- real-time communication/network protocols featuring randomisation (e.g. random choice of back-off etc.)

PRISM case studies:

- Firewire (IEEE 1394)
- CSMA/CD
- WiFi (802.11)
- ZigBee (802.15.4)
- Zeroconf

High-level modelling formalisms

- PRISM modelling language (DTMCs, CTMCs, MDPs)
  - simple, state-based language
  - based on Reactive Modules [Alur/Henzinger]
  - modules (system components, composed in parallel)
  - variables (finite-valued – integer ranges or booleans)
  - guarded commands (labelled with probabilities/rates)
  - parallel composition of modules – either asynchronous or synchronous (CSP-style)

- Translations to PRISM language from
  - process algebrae (PEPA, prob-CSP, probabilistic π-calculus)
  - graphical modelling for wireless networks [Fehnker et al.]

- Probmela – probabilistic extension of Promela

Property specifications

- Probabilistic extensions of temporal logic (CTL)
  - PCTL (for DTMCs/MDPs), CSL (for CTMCs), PTCTL (for PTAs)
  - essentially (timed) probabilistic reachability
- The P operator – quantitative extension of CTL's A and E
  - send → $P_{\geq 0.95}$ [F deliver] – "if a message is sent, then the probability of it eventually being delivered is at least 0.95"
  - for MDPs/PTAs, quantity over resolutions of nondeterminism, e.g. "probability of... is at least 0.95 for all possible schedulers"
- Interesting issues
  - no (simple) counterexamples, focus is on quantitative results
  - adding costs/rewards
  - combining quantitative and exhaustive analysis

Quantitative properties

- Model properties (and requirements) inherently quantitative
  - quality of service: how reliable is my car's Bluetooth network?
  - how efficient is my phone's power management policy?
  - quantifications of trust, anonymity, …
- How to generate counterexamples?
  - counterexample to CTL property $\neg$EF error is a finite trace
  - what is a good counterexample for $P_{<0.01}$ [F error] ?
  - some work on this [Han/Katoen, TACAS'07] [Aljazzar et al.]
  - can also generate best-/worst-case schedulers for MDPs
- So focus on quantitative properties...

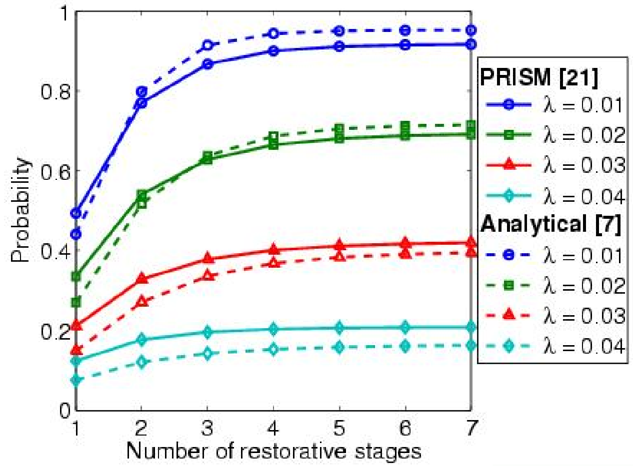
Quantitative properties

- Consider a PCTL formula $P_{<p}$ [F error]
  - if the probability is unknown, how to choose the bound p?
  - PRISM allows properties of the form $P_{=?}$ [F error]
  - (when the P is the outermost operator of the formula)
  - "what is the probability of reaching an error state?"
  - model checking is no harder: compute the values anyway
  - and for MDPs/PTAs: $P_{min=?}$ [F error] and $P_{max=?}$ [F error]

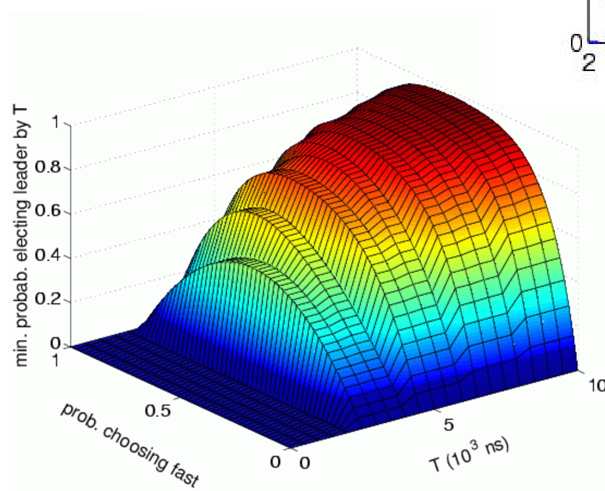
- Experiments: ranges of model/property parameters
  - e.g. $P_{=?}$ [$F^{\leq T}$ error] for T=1..100, N=1..5
  - identify patterns, trends, anomalies in quantitative results
  - investigate trade-offs, e.g. between...

Some real examples

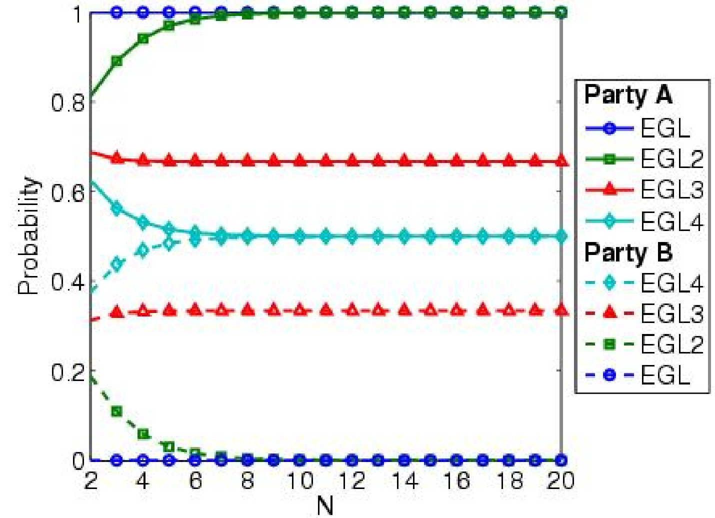
- NAND multiplexing system
  - $P_{=?}$ [F err/total>0.1]
  - “what is the probability that 10% of the NAND gate outputs are erroneous?”
- FireWire communication protocol
  - $z \,.\, P_{min=?}$ [F (z $\leq$ t) & (done_1 | done_2)]
  - “what is the minimum probability that a leader node has been elected before the clock reaches t?”
- Security: EGL contract signing protocol
  - $P_{=?}$ [F (pairs_a=0 & pairs_b>0)]
  - “what is the probability that the party B gains an unfair advantage during the execution of the protocol?”



Probability that 10% of gate outputs are erroneous for varying gate failure rates and numbers of stages



Optimum probability of leader election by time T for various coin biases



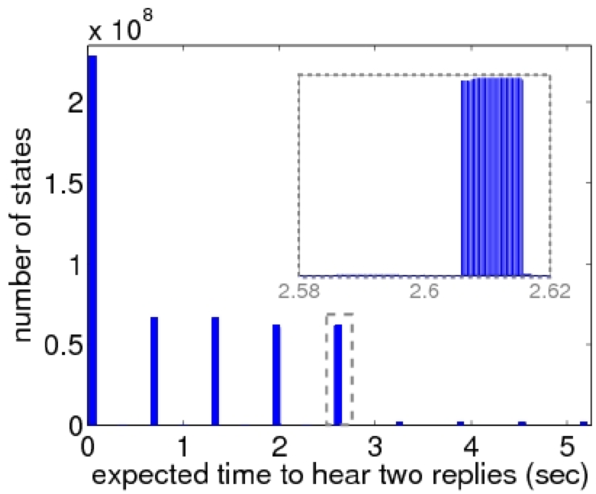
Probability that parties gain unfair advantage for varying numbers of secret packets sent

Costs and rewards

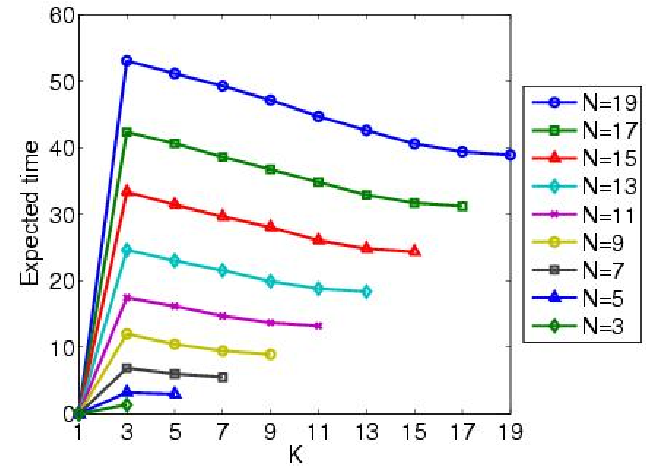
- Augment models with rewards (or, conversely, costs)
  - real-valued quantities assigned to states and/or transitions
  - no distinction between rewards ("good") and costs ("bad")
  - simple but flexible, many possible interpretations
- Some examples:
  - elapsed time, power consumption, size of message queue, number of messages successfully delivered, net profit, …
- Analyse expected value of these costs/rewards
  - instantaneous or cumulative interpretation, e.g.:
  - $R_{=?}$ [$I^{=t}$] e.g. "expected message queue size at time t?"
  - $R_{=?}$ [F end] e.g. "expected time for protocol termination?"
  - $R_{max=?}$ [$C^{\leq 2}$] e.g. "maximum expected power consumption during the first 2 hours that the system is in operation?"

Quantitative and exhaustive analysis

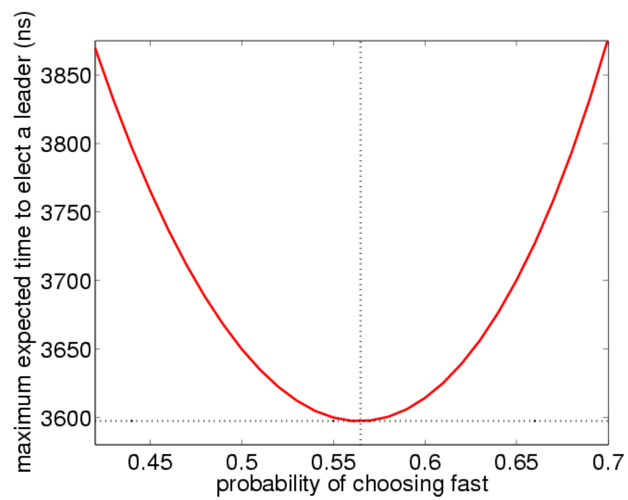
- Combining “quantitative” and “exhaustive” aspects
  - analysis of best/worst-case scenarios
- All possible resolutions of nondeterminism (MDPs)
  - $P_{min=?}$ [!end2 U end1] – “minimum probability of process 1 finishing before process 2, for any scheduling of processes?”
- Computing values for a range of states
  - $P_{=?}$ [$F^{\leq t}$ reply_count=k {“init”}{min}]
    “what is the minimum probability, from any initial configuration, that the sender has received k acknowledgements within t clock-ticks?”
  - $P_{=?}$ [$F^{\leq t}$ elected {tokens≤k}{min}] – “minimum probability of the leader election algorithm completing within t steps from any state where there are at most k tokens”



Bluetooth:
Distribution of expected time for two replies to be received, over all possible initial configurations of sender/receiver ($1.7x10^{10}$ states)



Self-stabilisation:
Worst-case expected number of steps to stabilise for initial configurations with K tokens amongst N processes



Firewire:
Maximum expected time for leader election for various coin biases

Probabilistic model checking

- Significant overlap between implementation of probabilistic model checking for DTMCs, CTMCs, MDPs:
  - graph-based algorithms on underlying transition system
    - reachability, qualitative probabilistic reachability
  - numerical computation – calculation of probabilities, rewards
    - usually, linear equation systems or linear optimisation problems
    - typically use iterative methods, e.g. Gauss-Seidel, value iteration
  - also: simulation-based sampling for approximate analysis
  - successful development of efficient symbolic implementations
    - see for example PRISM's MTBDD based engines
- For PTAs:
  - probabilistic extensions of: region graph approach, forwards/backwards symbolic reachability, digital clocks

Tool support

- Tool support provides:
  - high-level languages/formalisms, automation of verification and experiments, efficient (e.g. symbolic) implementations, discrete event simulators for debugging/approximations
- PRISM: symbolic model checking of DTMCs, CTMCs, MDPs
  - Kwiatkowska, Norman, Parker, ... at Birmingham/Oxford
  - indirect support for PTAs (direct translation using digital clocks, combine with Kronos/prototype reachability implementations)
- ETMCC/MRMC: DTMCs, CTMCs + reward extensions
- LiQuor: LTL verification for MDPs (Probmela language)
- RAPTURE: prototype for abstraction/refinement of MDPs
- And more: APMC, Ymer, VESTA, APNN-Toolbox, SMART, CADP, Möbius

Current research directions

- State space explosion problem
  - scalability is always an important issue
  - as for non-probabilistic (and other) verification techniques
  - current work addressing this includes...
- Symmetry reduction
  - exploitation of multiple identical components
- Compositionality
  - combining analysis results for individual components
- Abstraction/refinement
  - reduction of large/infinite systems (e.g. software verification)
- And others:
  - e.g. symbolic (BDD-based) implementations, partial order reduction, ...

Symmetry reduction

- Fully symmetric systems (N identical components)
  - quite common (e.g. symmetric communication protocols)
  - potentially exponential reduction in state space
  - resulting quotient model = strong probabilistic bisimulation
- Two approaches
  - counter abstraction – language level translation of model [Donaldson/Miller], GRIP
  - symbolic (MTBDD) implementation – reduction of full model to quotient based on state sorting [KNP, CAV'06]
- Also: algorithms to compute (full) bisimulation reduction
  - including symbolic (MTBDD) implementations [Derisavi]

Compositionality

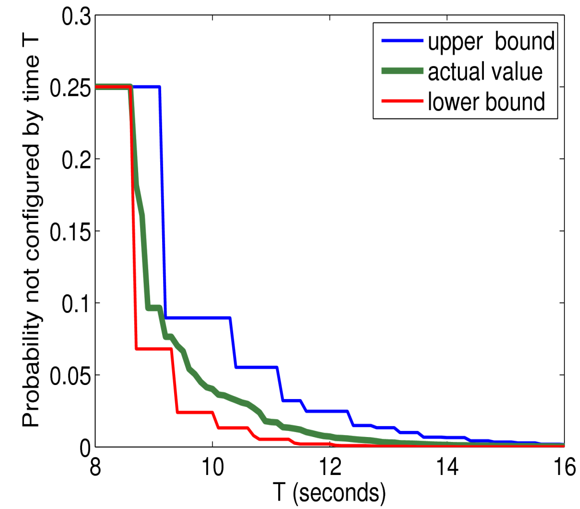
- How to apply to probabilistic model checking?
  - parallel composition of components $M_1||M_2$ is nondeterministic
  - analysis of $M_1||M_2$ is of the form "for any (history dependent) scheduler, the probability of $M_1||M_2$ satisfying ϕ is..."
  - introduces causal dependencies between components
- Possible solutions include:
  - partial information schedulers [de Alfaro/Henzinger/...]
  - restricted class of schedulers, e.g. based on token passing, partial information [Cheung]
  - multi-objective criteria for MDPs [Etessami et al., TACAS'07]
- What about quantitative results and compositionality?

Abstraction and Refinement

- Construct abstract model with unimportant info discarded

- What form does the abstract model take?
  - abstract model is "more nondeterministic"
  - DTMCs/MDPs
    - → MDPs [D'Argenio et al.]
    - → DTMCs + intervals [Fecher et al.]
    - → 2-player stochastic games [KNP, QEST'06]
  - CTMCs -> CTMDPs [Katoen et al.]

- How to generate/refine abstractions?
  - predicate abstraction? [Wachter et al.]
  - counterexample guided refinement?



Analysis of Zeroconf using 2-player games [KNP, QEST'06]

Summary

- Probabilistic model checking
  - automated verification of exact quantitative results for a wide range of models and properties
  - efficient implementations and tool support available
  - successfully deployed in wide range of application domains
  - many more challenges remain...
- For more information, see the PRISM web page
  - www.prismmodelchecker.org
  - case studies, related publications, lectures, talks, tutorials, links, tool download/documentation